

18.703 HOMEWORK #6, DUE TUESDAY APRIL 3RD

1. Find a presentation of the dihedral group D_n .
2. Herstein, Chapter 2, §9, 2.
3. Herstein, Chapter 3, §3, 1.
4. Herstein, Chapter 3, §3, 2.
5. Herstein, Chapter 3, §3, 3.
6. Herstein, Chapter 3, §3, 6: Show that the 3-cycles generate A_n .
7. Herstein, Chapter 3, §3, 7.
8. Let P be a Sylow p -subgroup of a finite group G . Show that P is characteristically normal in G if and only if there is only one Sylow p -subgroup.
9. Let G be an abelian group of order $p^l q^m$, where p and q are distinct primes and $l > 0$, $m > 0$. Show that $G \simeq P \times Q$, where P is a subgroup of order p^l and where Q is a subgroup of order q^m .
10. Let G be a simple group of order 168. Show that G is isomorphic to a subgroup of S_8 .
11. Show that every simple group whose order lies between 1 and 59 has prime order.
12. **Challenge Problem:** Herstein, Chapter 3, §3, 9 & 10:
 - (i) If $n \geq 5$ and $\{e\} \neq N \triangleleft A_n$ is a normal subgroup, show that N must contain a 3-cycle.
 - (ii) Show that A_n is simple if and only if $n \neq 4$.
13. **Challenge Problem:** Show that every simple group whose order lies between 61 and 167 has prime order.