18.703 HOMEWORK #3, DUE THURSDAY MARCH 7TH

1. Herstein, Chapter 3, §1, 1 & 5: Find the products and their orders:
(i)
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 5 & 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 1 \end{pmatrix}.$$
(ii)
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 4 & 5 \end{pmatrix}.$$
(iii)
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 3 & 2 & 5 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 3 & 2 & 5 \end{pmatrix}.$$
2. Herstein, Chapter 3, §2, 1.
3. Herstein, Chapter 3, §2, 2.
4. Herstein, Chapter 3, §2, 3 (a), (f).
5. Find the conjugate of $\sigma = (1, 4, 7, 2)(3, 6, 5) \in S_7$ by $\tau = (1, 2, 3)(4, 7, 5)$. What is the order of σ and τ?
6. Find an element $\tau \in S_7$ that carries $\sigma = (1, 2, 5)(3, 6, 7, 4)$ into $\sigma' = (3, 1, 4)(2, 7, 6, 5)$, that is, find $\tau \in S_7$ such that
$$\sigma' = \tau\sigma\tau^{-1}.$$
7. Herstein, Chapter 3, §2, 17.