PRACTICE FINAL B
MATH 18.02, MIT, AUTUMN 12

You have three hours. This test is closed book, closed notes, no calculators.

There are 16 problems, and the total number of points is 240. Show all your work. *Please make your work as clear and easy to follow as possible.*

Name:____________________

Signature:____________________

Student ID #:____________________

Recitation instructor:____________________

Recitation Number+Time:____________________

Problem	Points	Score
1	15	
2	10	
3	10	
4	10	
5	15	
6	20	
7	15	
8	15	
9	15	
10	15	
11	15	
12	15	
13	15	
14	15	
15	20	
16	20	
Total	240	

1. (15pts) (i) Let $A = (1, 2, 3)$, $B = (4, -1, 4)$ and $C = (2, 4, 6)$. Find the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(ii) Let $P = (0, 1, 1)$, $Q = (2, 1, 0)$ and $R = (1, 3, 2)$. Find the cross product of $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

(iii) Find the equation of the plane containing P, Q and R.

2. (10pts) At what point does the line through $(1, 0, -1)$ and $(2, 1, 2)$ intersect the plane $x + y - z = 3$?

3. (10pts) Give parametric equations for the line given as the intersection of the two planes $2x - y - 4z = 0$ and $5x - 2z = 1$.

4. (10pts) A moon M revolves around a planet P in a circular orbit of radius one in the xy-plane, so that in one year it completes two revolutions. Meanwhile the planet revolves around a star O in a circular orbit of radius five, one revolution per year. The star is always at the origin and at time $t = 0$ the planet and the moon are on the x-axis, the planet to the right of the sun and the moon to the right of the planet. Find the position of the moon as a function of the number of years t.

5. (15pts) Let S be the surface defined by the equation

$$z = x^2y + xy^2 - 3y^2.$$

(i) Find the tangent plane to S at the point $P = (2, 1, 3)$.

(ii) Give a formula approximating the change Δz in z if x and y change by small amounts Δx and Δy.

(iii) Approximate the value of z at the point $(x, y) = (2.01, 1.01)$.

6. (20pts) A rectangular box lies in the first quadrant. One vertex is at the origin and the diagonally opposite vertex P is on the plane $2x+y+z=2$. We want the coordinates of the point P which maximises the volume of the box.

(i) Show that this lead to maximising the function

$$f(x,y) = xy(2-2x-y).$$

Find the critical points of $f(x,y)$.

(ii) Determine the type of the critical point in the first quadrant.

(iii) Now solve this problem using the method of Lagrange multipliers.

7. (15pts) Find the point on the surface

$$z^2 = xy + x + 1$$

closest to the origin, using the method of Lagrange multipliers.

8. (15pts) Let $w(x, y, z) = x^4 + 2xy^2 - z^3$.
(i) Find the equation of the tangent plane to the surface $w = 2$ at $(1, 1, 1)$.

(ii) Assume that x, y and z are constrained by the equation $w(x, y, z) = 2$. Find the value of

$$\left(\frac{\partial x}{\partial z}\right)_y$$

at $(1, 1, 1)$.

9. (15pts) Let R be the plane triangle with vertices $(0,0)$, $(1,-1)$ and $(1,1)$. Set up an iterated integral which gives the average distance of a point from the origin,

(i) in rectangular coordinates

(ii) in polar coordinates.

10. (15pts) Let C_1 be the line segment from $(0,0)$ to $(1,0)$, C_2 the arc of the unit circle running from $(1,0)$ to $(0,1)$ and let C_3 be the line segment $(0,1)$ to $(0,0)$. Let C be the simple closed curve formed by C_1, C_2 and C_3 and let

$$\vec{F} = x^3\hat{\imath} + x^2 y\hat{\jmath}.$$

Calculate

$$\oint_C \vec{F} \cdot \mathrm{d}\vec{r},$$

(i) directly.

(ii) using Green's theorem.

11. (15pts) (i) Calculate the flux of $\vec{F} = x\hat{\imath}$ out of each side, S_1, S_2, S_3 and S_4 of the square $-1 \leq x \leq 1$, and $-1 \leq y \leq 1$. Label the sides so that S_1 and S_3 are horizontal, S_1 below S_3, and S_2 and S_4 are vertical, S_2 to the right of S_4.

(ii) Explain why the total flux out of any square of sidelength 2 is the same, regardless of its location or how its sides are tilted.

12. (15pts) Find the area of the region R bounded by the curves $xy = 2$, $xy = 5$, $y = x^2$ and $y = 4x^2$.

13. (15pts) Let

$$\vec{F} = (y - z)\hat{\imath} + (x + y)\hat{\jmath} + (1 - x)\hat{k}$$

(i) Find a potential function f for $\vec{F}$.

(ii) Let C be the parametric curve

$$x = 3\cos^3 t \qquad y = 3\sin^3 t \qquad z = t \qquad \text{for} \qquad 0 \le t \le 2\pi.$$

Find

$$\int_C \vec{F} \cdot \mathrm{d}\vec{r}.$$

14. (15pts) Let D be the portion of the solid sphere

$$x^2 + y^2 + z^2 < 1,$$

lying above the plane

$$z = \frac{\sqrt{2}}{2}.$$

The surface bounding D consists of two parts, a curved part S and flat part T. Orient both surfaces so that the normal vector points upwards. Let

$$\vec{F} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}.$$

(i) Calculate the flux of $\vec{F}$ across S.

(ii) Calculate the flux of $\vec{F}$ across T.

(iii) Find the volume of D using the divergence theorem.

15. (20pts) Calculate the flux of

$$\vec{F} = x\hat{\imath} + y\hat{\jmath} + (1 - 2z)\hat{k}$$

out of the solid bounded by the xy-plane and the paraboloid $z = 4 - x^2 - y^2$.

(i) directly,

(ii) using the divergence theorem.

16. (20pts) Let $\vec{F} = -y\hat{\imath} + x\hat{\jmath}$ and let S be the surface of the hemisphere

$$x^2 + y^2 + (z-1)^2 = 1 \qquad \text{and} \qquad z \geq 1,$$

oriented updwards.

(i) Calculate the flux of $\vec{F}$ across S.

(ii) Find the curl of $\vec{F}$.

(iii) Calculate the flux of $\operatorname{curl} \vec{F}$ across S using Stokes' theorem.