THIRD PRACTICE MIDTERM B
MATH 18.02, MIT, AUTUMN 12

You have 50 minutes. This test is closed book, closed notes, no calculators.

There are 6 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.*

Name:____________________________

Signature:____________________________

Student ID #:____________________________

Recitation instructor:____________________________

Recitation Number+Time:____________________________

Problem	Points	Score
1	15	
2	15	
3	10	
4	20	
5	20	
6	20	
Total	100	

1. (15pts) Let $(\bar{x}, \bar{y})$ be the centre of mass of the triangle with vertices at $(-2, 0)$, $(0, 1)$ and $(2, 0)$ (assume uniform density $\delta = 1$).
(i) Express $\bar{y}$ in terms of an integral.

(ii) Find $\bar{x}$.

2. (15pts) Find the moment of inertia about the origin of the half disk $x^2 + y^2 < a^2$, $x > 0$, where the density $\delta = x^2$.

3. (10pts) For which value of a is the vector field

$$\vec{F} = \left(axy - \frac{1}{x}\right)\hat{\imath} + \left(x^2 - \frac{1}{y}\right)\hat{\jmath}$$

a gradient vector field?

4. (20pts) Let $\vec{F} = 2y\hat{\imath} - x\hat{\jmath}$. Let C be the curve $y = x^2$ starting at $(0, 0)$ and ending at $(1, 1)$.

(a) Compute the work done on a particle that moves along C.

(b) Compute the flux of $\vec{F}$ across C.

5. (20pts) (i) Express the work done by the force field

$$\vec{F} = (5x + 3y)\hat{\imath} + (1 + \cos y)\hat{\jmath}$$

on a particle going counterclockwise once around the unit circle centred at the origin in the form

$$\int_a^b f(t)\,\mathrm{d}t.$$

(ii) Evaluate the work done by using Green's theorem.

6. (20pts) Consider the region R enclosed by the x-axis, $x = 1$ and $y = x^3$. Let $\vec{F} = (1 + y^2)\hat{\jmath}$.
(i) Use the normal form of Green's theorem to find the flux of $\vec{F}$ out of R.

(ii) Find the flux across the horizontal side C_1 of R and the vertical side C_2 of R.

(iii) Find the flux across the third side C_3.