THIRD PRACTICE MIDTERM A
MATH 18.02, MIT, AUTUMN 12

You have 50 minutes. This test is closed book, closed notes, no calculators.

There are 6 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.*

Name:_____

Signature:_____

Student ID #:_____

Recitation instructor:_____

Recitation Number+Time:_____

Problem	Points	Score
1	15	
2	10	
3	20	
4	20	
5	20	
6	15	
Total	100	

1. (15pts) Evaluate

$$\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \frac{1}{1-y}\,\mathrm{d}y\,\mathrm{d}x.$$

2. (10pts) Find the x-coordinate $\bar{x}$ of the centre of mass of the portion of the unit disk in the first quadrant, bounded by the x-axis and the line $y = x$ (assume the density $\delta = 1$).

3. (20pts) (i) Find a such that

$$\vec{F} = (x^2 - 14xy)\hat{\imath} + (ax^2 - 2y)\hat{\jmath}.$$

is conservative.

(ii) Find a potential function $f(x, y)$ for $\vec{F}$ for this value of a.

(iii) Calculate the line integral

$$\int \vec{F} \cdot \mathrm{d}\vec{r},$$

for the curve C given by $x = 3\sin t$, $y = \cos t$, $0 \leq t \leq \pi$, where C has the orientation which starts at $(0, 1)$.

4. (20pts) Consider the region in the xy-plane bounded by the curves $y = x^2$, $y = x^2/3$, $xy = 3$ and $xy = 5$.
(i) Compute $\mathrm{d}u\,\mathrm{d}v$ in terms of $\mathrm{d}x\,\mathrm{d}y$, where $u = x^2/y$ and $v = xy$.

(ii) Find a double integral for the area of R in uv-coordinates and evaluate it.

5. (20pts) Let C be the positively oriented closed curve formed by the parabola $y = x^2$ running from $(-1, 1)$ to $(1, 1)$ and by a horizontal line segment running from $(1, 1)$ back to $(-1, 1)$. Evaluate

$$\oint_C y^2 \, \mathrm{d}x + 4xy \, \mathrm{d}y,$$

(i) directly, and

(ii) by using Green's theorem.

6. (15pts) Find the volume of the region enclosed by the plane $z = 4$ and the surface

$$z = (2x - y)^2 + (x + y - 1)^2.$$