SECOND PRACTICE MIDTERM B
MATH 18.02, MIT, AUTUMN 12

You have 50 minutes. This test is closed book, closed notes, no calculators.

There are 6 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.*

Name:_____

Signature:_____

Student ID #:_____

Recitation instructor:_____

Recitation Number+Time:_____

Problem	Points	Score
1	15	
2	20	
3	20	
4	15	
5	15	
6	15	
Total	100	

1. (15pts) Two sides of a triangle are a and b and θ is the angle between them. The third side is c.
(i) Write down an approximation for Δc in terms of a, b, c, θ, Δa, Δb and Δc.

(ii) If $a = 1$, $b = 2$, $\theta = \pi/3$, is c more sensitive to changes in Δa or Δb?

2. (20pts) A particle is moving in the xy-plane so that relative to the origin it is rotating clockwise at a rate of 3 radians per second while its distance to the origin is increasing at a rate of 7 metres per second. At a time when the particle is at $(5, -12)$, what is

$$\frac{dx}{dt}?$$

3. (20pts) Find the global maximum and minimum of

$$f(x, y) = x^2 - xy + y^2 + 4$$

over the triangle with sides $x = 0$, $y = 4$ and $y = x$.

4. (15pts) Let $u = x^2 - y^2$, $v = 2xy$ and $w = w(u, v)$.
(i) Express the partial derivatives w_x and w_y in terms of w_u and w_v (and x and y).

(ii) Assume that w solves the differential equation $uw_u + vw_v = 1$. Find $xw_x + yw_y$.

5. (15pts) (i) Write down the equations for the Lagrange multiplier for the point of the surface

$$x^4 + y^4 + z^4 + yz + xz + xy = 6$$

with the largest x-value.

(ii) If the point with the largest x-value is (x_0, y_0, z_0) find the equation of the tangent plane at this point.

6. (15pts) Suppose that $x^2y + xz^2 = 1$ and let $w = x^3y$. Express

$$\left(\frac{\partial w}{\partial z}\right)_y,$$

as a function of x, y and z and determine its value when $(x, y, z) = (1, 1, 2)$.