SECOND PRACTICE MIDTERM A
MATH 18.02, MIT, AUTUMN 12

You have 50 minutes. This test is closed book, closed notes, no calculators.

There are 6 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.*

Name: ____________________

Signature: ____________________

Student ID #: ____________________

Recitation instructor: ____________________

Recitation Number+Time: ____________________

Problem	Points	Score
1	20	
2	10	
3	20	
4	20	
5	15	
6	15	
Total	100	

1. (20pts) Let p be the point on the curve $x^2 + x^3 - y^2 = 3.1$ which is closest to $(2, 3)$. Use the gradient to estimate the coordinates of p.

2. (10pts) Find the equation of the tangent plane to the surface

$$x^2 + 3y^2 + 2z^2 = 12 \qquad \text{at the point} \qquad (1, -1, 2).$$

3. (20pts) (i) Find the critical points of

$$w = f(x, y) = 5x^2 - 2xy + 2y^2 - 8x - 2y + 7,$$

and determine their type.

(ii) Find where $f(x, y)$ is smallest in the first quadrant, $x \geq 0$ and $y \geq 0$. Justify your answer.

4. (20pts) Using Lagrange multipliers, find the points on the ellipse $x^2 + 2y^2 = 1$ where the function $f(x, y) = xy$ has a maximum and a minimum.

(i) Write down the equations satisfied by the Lagrange multiplier.

(ii) Solve these equations and find the global maximum and minimum.

5. (15pts) Given that the variables w, x, y and z satisfy $w = xyz$ and $w^2 + z^2 = 13$, find

$$\left(\frac{\partial w}{\partial x}\right)_y,$$

when $w = 3$, $x = 3$, $y = 1/2$ and $z = 2$.

6. (15pts) The two surfaces $x^4 - y^3 + z^2 = 2$ and $x^2y^2 + 3z^4 = 4$ intersect along a curve for which y is a function of x. Find

$$\frac{dy}{dx} \qquad \text{at} \qquad (x_0, y_0, z_0) = (1, 1, 1).$$