18.02 HOMEWORK #3, DUE THURSDAY SEPTEMBER 27TH

Part A (17 points)

(09/20) Read: 13.2, 13.4; Notes TA.
13.2/7, 15, 53–58
2A/1abcde, 2abde, 3abc, 5ab
2B/1ab, 3, 6, 9;
13.4/57.
(09/21) Holiday
(09/25) Review

Part B (15 points)

1. (Immediately, 8 points: 2+1+1+2+2) In 3D computer graphics, one needs to represent 3-dimensional objects on a plane screen, by drawing a given point P at the place where the line from P to the eye meets the screen. Suppose that the screen is the yz-plane, and the eye is at $E = (2, 0, 0)$.
(i) At what point $Q = (y, z)$ in the yz-plane should one represent the point $P = (x_0, y_0, z_0)$? (Express y and z in terms of the coordinates of P. Assume that $x_0 < 2$. Why is this assumption legitimate?)
(ii) What does the image on the screen of a line segment in space look like? (Justify your answer.)
(iii) A line segment connects $P_0 = (-1, -3, 1)$ to $P_1 = (-2, 4, 6)$. What is drawn on the screen?
(iv) A bird leaves from P_0 at time $t = 0$, and flies in a straight line at constant speed in such a way that it passes through P_1 at time $t = 1$. What does the trajectory of the bird (for $t \geq 0$) look like on the screen? Show that, as t tends to infinity, the trajectory on the screen tends to a limit point (the "vanishing point"), and give its coordinates.
(v) In fact, part of the trajectory of the bird is hidden by a vertical fence erected in front of the observer. The fence lies in the plane $x = 1$, and its top is at the altitude $z = 1$. What portion of the trajectory is hidden? (*Hint: observe that the points hidden by the fence are exactly those which lie below a certain plane passing through E.*)
2. (Thursday, 3 points: 1+2)
(i) Sketch some level curves of the function $\max(|x|, |y|)$.
(ii) Find a function whose level curves are the same as in (a), but rotated through $\pi/4$.
3. (Thursday, 4 points: 3+1)
(i) Let $w = x^y (= e^{y \ln x})$. Give an approximate formula for the small change Δw in the value of w produced by small changes Δx and Δy in the values of x and y. Use this to calculate an approximate value of $1.98^{2.01}$. Compare your answer with the exact value.

b) Starting from $2^2 = 4$, is the value of w more sensitive to small changes in x or in the exponent y? (Justify your answer.)