HWK #4, DUE WEDNESDAY 03/14

1. Let $F' \subset \mathbb{R}^3$ be the fan whose one dimensional cones are generated by

$$\begin{array}{lll} v_1 = (1,0,0) & v_2 = (0,1,0) & v_3 = (0,0,1) \\ v_4 = (0,-1,-1) & v_5 = (-1,0,-1) & v_6 = (-2,-1,0). \end{array}$$

and whose maximal cones are

$$\begin{array}{llll} \langle v_1, v_2, v_3 \rangle & \langle v_1, v_2, v_4 \rangle & \langle v_2, v_4, v_5 \rangle & \langle v_2, v_3, v_5 \rangle \\ \langle v_3, v_5, v_6 \rangle & \langle v_1, v_3, v_6 \rangle & \langle v_1, v_4, v_6 \rangle & \langle v_4, v_5, v_6 \rangle. \end{array}$$

Let F be the fan obtained from F' by inserting the vectors $v_7 = (-1,-1,-1)$ and $v_8 = (-2,-1,-1)$ and subdividing accordingly.
(i) Show that X is a smooth toric variety.
(ii) Show that if $D = \sum a_i D_i$ is a base point free T-Cartier divisor then $D \sim 0$. (*Hint: focus on the three cones* $\langle v_1, v_2, v_4 \rangle$, $\langle v_2, v_3, v_5 \rangle$ *and* $\langle v_1, v_3, v_6 \rangle$*.*)
(iii) Show that X is not a projective variety.
2. Hartshorne: Chapter II, 7.8-7.12. (Assume that X is connected in (II.7.9). Change the condition in (II.7.12) to the statement that neither one contains an irreducible component of the other to avoid trivial counterexamples).
3. **Challenge Problem** Chapter II, 7.13.