# HWK #12, DUE WEDNESDAY 5/4

1. Consider the versal deformation space of the tacnode, $y^2 + x^4$. Describe the locus of points which correspond to $A_1$ and $A_2$ singular points.

2. Let $C \subset \mathbb{A}^2$ be a curve. Show that if $C$ has a tacnode then $C$ contains a zero dimensional scheme isomorphic to the scheme

$$z = \mathrm{Spec}\,\frac{\mathbb{C}[x,y]}{\langle y, x^2 \rangle^2}.$$

What about for a singularity of type $A_{2n+1}$?

3. Let $\pi\colon T \longrightarrow S$ be a resolution of singularities of a normal surface $S$. The **resolution graph** of $\pi$ has vertices corresponding to the prime exceptional divisors of $\pi$ and an edge between two vertices if and only if the two prime divisors intersect.

Show, by explicit calculation, that the $ADE$-singularities have a resolution graph of type the corresponding Dynkin diagram.

4. Find the resolution graph of a singularity of type $1/13(1,7)$ and of type $1/15(1,4)$.

5. Let $X$ be a toric variety. Show that there is a resolution of singularities $\pi\colon Y \longrightarrow X$ which is toric (so that $Y$ is a toric variety and $\pi$ is a toric morphism).