PRACTICE FINAL
MATH 18.022, MIT, AUTUMN 10

You have three hours. This test is closed book, closed notes, no calculators.

Name:________________________

Signature:________________________

Recitation Time:________________________

There are 10 problems, and the total number of points is 200. Show all your work. *Please make your work as clear and easy to follow as possible.*

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
Total	200	

1. (20pts) Find the shortest distance between the line l given parametrically by

$$(x, y, z) = (1 + 2t, -3 + t, 2 - t),$$

and the intersection of the two planes Π_1 and Π_2 given by the equations

$$x + y + z = 3 \qquad \text{and} \qquad x - 2y + 3z = 2.$$

2. (20pts) Let W be the solid inside the sphere $x^2 + y^2 + z^2 = 9$ and above the top sheet of the hyperboloid $x^2 + y^2 - z^2 = -1$.

(a) Set up an integral in cylindrical coordinates for evaluating the volume of W.

(b) Evaluate this integral.

3. (20pts) Evaluate

$$\int_0^{\pi/2} \left(\int_y^{2y} \frac{\sin x}{x} \, \mathrm{d}x \right) \mathrm{d}y + \int_{\pi/2}^{\pi} \left(\int_y^{\pi} \frac{\sin x}{x} \, \mathrm{d}x \right) \mathrm{d}y.$$

4. (20pts) Let $\vec{r}\colon I \longrightarrow \mathbb{R}^3$ be a regular smooth curve parametrised by arclength. Let $a \in I$ and suppose that

$$\vec{T}(a) = \frac{4}{9}\hat{\imath} - \frac{7}{9}\hat{\imath} - \frac{4}{9}\hat{k}, \quad \vec{N}(a) = \frac{1}{9}\hat{\imath} - \frac{4}{9}\hat{\imath} + \frac{8}{9}\hat{k}, \quad \frac{d\vec{N}}{ds}(a) = -\frac{4}{3}\hat{\imath} + \frac{1}{3}\hat{\jmath} + \frac{1}{3}\hat{k}.$$

Find:

(i) the unit binormal vector $\vec{B}(a)$.

(ii) the curvature $\kappa(a)$.

(iii) the torsion $\tau(a)$.

5. (20pts) Let

$$S = \{ (x, y, z) \in \mathbb{R}^3 \mid xy^2z^3 - x^2y^2z^2 + x^3y^3 = 1 \}.$$

(a) Show that in a neighbourhood of the point $P = (1, 1, 1)$, the subset S is the graph of a smooth function $z = f(x, y)$.

(b) Find the derivative $Df(1, 1)$.

6. (20pts) Let K be the solid bounded by the four planes $x = 0$, $y = 0$, $z = 0$ and $x + 2y + 3z = 12$ and let $f\colon K \longrightarrow \mathbb{R}$ be the function $f(x, y, z) = xyz$.

(a) Show that f has a global maximum on K.

(b) Find this global maximum value of f on K.

7. (20pts) Let D be the region bounded by the four curves $xy = 1$, $xy = 3$, $2y = x^2$ and $y = x^2$.

(a) Compute $\mathrm{d}x\,\mathrm{d}y$ in terms of $\mathrm{d}u\,\mathrm{d}v$, where $u = xy$ and $v = x^2/y$.

(b) Find the area of D.

8. (20pts) Find the line integral of the vector field

$$\vec{F}(x,y) = \frac{-y}{x^2+y^2}\hat{\imath} + \frac{x}{x^2+y^2}\hat{\jmath},$$

along the following oriented curves:
(i) The circle C_1 with equation $x^2+y^2=1$, oriented counter-clockwise.

(ii) The ellipse C_2 with equation $x^2+2y^2=4$, oriented counter-clockwise.

9. (20pts) Find the surface scalar integral

$$\iint_S (x^2 + y^2)\,\mathrm{d}S,$$

where S is the sphere of radius a centred at the origin.

10. (20pts) A smooth vector field is defined on the whole of $\mathbb{R}^3$, except two lines L and M, which intersect at the point P. Suppose that $\operatorname{curl} \vec{F} = 0$, and that

$$\int_{C_1} \vec{F} \cdot \mathrm{d}\vec{s} = -2, \qquad \int_{C_2} \vec{F} \cdot \mathrm{d}\vec{s} = 3 \qquad \text{and} \qquad \int_{C_3} \vec{F} \cdot \mathrm{d}\vec{s} = 1.$$

Find the following integrals. Give your reasons.

(a)

$$\int_{C_4} \vec{F} \cdot \mathrm{d}\vec{s}.$$

(b)

$$\int_{C_5} \vec{F} \cdot \mathrm{d}\vec{s}.$$