THIRD PRACTICE MIDTERM
MATH 18.022, MIT, AUTUMN 10

You have 50 minutes. This test is closed book, closed notes, no calculators.

Name:_____

Signature:_____

Recitation Time:_____

There are 5 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.*

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20pts) For what values of λ, μ and ν does the function $f\colon \mathbb{R}^3 \longrightarrow \mathbb{R}$,

$$f(x, y, z) = \lambda x^2 + \mu xy + y^2 + \nu z^2,$$

have a non-degenerate local minimum at $(0, 0, 0)$?

2. (20pts) Let $f\colon \mathbb{R}^3 \longrightarrow \mathbb{R}$ be the function $f(x, y, z) = 2x + y - z$
(i) Show that f has a global minimum on the ellipsoid $x^2+2y^2+3z^2 = 6$.

(ii) Find this minimum.

3. (20pts)
(i) Draw a picture of the region of integration of

$$\int_0^1 \int_{1+x}^{\sqrt{9-x^2}} \mathrm{d}y\,\mathrm{d}x.$$

(ii) Change the order of integration of the integral.

4. (20pts) Let W be the region inside the two cylinders $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$.
Set up an integral to calculate the volume of W and calculate this integral.

5. (20pts) Let D be the region in the first quadrant bounded by the curves $y^2 = x$, $y^2 = 2x$, $xy = 1$ and $xy = 4$.
(i) Find $\mathrm{d}u\,\mathrm{d}v$ in terms of $\mathrm{d}x\,\mathrm{d}y$, where $u = \frac{y^2}{x}$ and $v = xy$.

(ii) Set up an integral to calculate the area of the region D and calculate this integral.