

**THIRD HOMEWORK, DUE THURSDAY SEPTEMBER
30TH**

Feel free to work with others, but the final write-up should be entirely your own and based on your own understanding.

1. (10 pts) Let L be the line which passes through the point $(a, 0, 0)$ and is parallel to the z -axis. Let D be the region that lies inside the cylinder of radius a centred around the line L and that lies between the planes $z = -1$ and $z = 3$. Describe the region D in cylindrical coordinates.

2. (10 pts) Let D be the region inside the sphere of radius $2a$ centred at the origin and that lies between the planes $x = -a$ and $x = a$. Describe the region D in spherical coordinates.

3. (15 pts) Suppose that $f: A \rightarrow B$ and $g: B \rightarrow C$ are two functions, and let $g \circ f: A \rightarrow C$ be their composition. For each statement below, say whether the statement is true or false. If true, give a reason and if false give a counterexample.

(i) If f and g are surjective, then $g \circ f: A \rightarrow C$ is surjective.

(ii) If $g \circ f: A \rightarrow C$ is surjective, then f is surjective.

(iii) If $g \circ f: A \rightarrow C$ is surjective, then g is surjective.

4. (10 pts) Let $S \subset \mathbb{R}^3$ be the right angled cone, with vertex at the origin and centred around the z -axis, and which lies on or above the xy -plane. Write down a function $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ such that $S = f^{-1}(c)$ is the level set of f at height c .

5. (10 pts) (2.1.34).

6. (5 pts) (2.2.9).

7. (5 pts) (2.2.11).

8. (5pts) (2.2.13).

9. (5 pts) (2.2.15).

10. (5 pts) (2.2.31).

11. (5 pts) (2.2.35).

12. (5 pts) (2.2.42).

Just for fun: What is the volume of the intersection of three cylinders of radius r and height h ? Assume that the cylinders are centred around the three coordinate axes and that the central point of each cylinder is the origin.