HWK #2, DUE WEDNESDAY 09/23

1. Let $p_1, p_2, \dots, p_{n+2}$ and $q_1, q_2, \dots, q_{n+2}$ be two sets of $n+2$ points in linear general position in $\mathbb{P}^n$. Show that there is a unique element of $\mathrm{PGL}(n+1)$ sending p_i to q_i.
2. Let K be an algebraically closed field. Show that, up to conjugacy, any element ϕ of $\mathrm{PGL}(2, K)$ is one of

   (1) the identity,
   (2) $z \longrightarrow az$, $a \in K^*$,
   (3) $z \longrightarrow z + 1$,

and that the three cases are distinguished by the number of fixed points; at least three; two; one.
3. Show that the twisted cubic is defined by the equations $XW = YZ$, $Y^2 = XZ$ and $Z^2 = YW$.
4. a) Show the intersection of any two of the quadrics above is the union of C and a line (in fact either a tangent line or a secant line, that is a line which meets C twice).
b) More generally, if $\lambda = [\lambda_0 : \lambda_1 : \lambda_2]$ is a point of $\mathbb{P}^2$, let F_λ denote the quadratic polynomial

$$\lambda_0(Y^2 - XZ) + \lambda_1(XW - YZ) + \lambda_2(Z^2 - YW).$$

Show that if $\lambda \neq \mu$ then the zero locus of F_λ and F_μ is also the union of C and a line (again, in fact either a tangent or secant line).
5. Show that any set of points on a rational normal curve are in linear general position.
6. Show that the image of $\mathbb{P}^n$ under the d-uple embedding is defined by the equations

$$Z_I Z_J = Z_{I'} Z_{J'},$$

where I, J, I' and J' are any $(n+1)$-tuples of positive integers who sum is d and

$$I + J = I' + J'.$$