

FIFTH HOMEWORK, DUE THURSDAY MARCH 20TH

1. Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two morphisms of schemes, and let $h = g \circ f: X \rightarrow Z$ be their composition.
 - (i) If f and g are closed immersions then show that h is a closed immersion.
 - (ii) If f and g are projective morphisms then show that h is a projective morphism.
2. Let k be a field.
 - (i) If $X \subset Y$ are schemes over k then show that $X(k) \subset Y(k)$.
 - (ii) If $X = \cup X_i$, where X and X_1, X_2, \dots, X_m are schemes over k then $X(k) = \cup X_i(k)$.
 - (iii) $\mathbb{A}_k^n(k) = k^n$.
 - (iv) $\mathbb{P}_k^n(k) = (k^{n+1} - \{0\})/k^*$.
3. Classify all irreducible zero dimensional schemes z of length three and four over an algebraically closed field k of characteristic zero.
4. Does your answer to (3) change if either the groundfield k is of characteristic p or k is not algebraically closed?
5. **Challenge Problems** Classify all irreducible zero dimensional schemes z of length five, over an algebraically closed field k .