

LECTURE 14: PROOF OF HUREWICZ

Proposition 0.1. The Hurewicz homomorphism induces a map of long exact sequences

$$\begin{array}{ccccccc} \cdots & \longrightarrow & \pi_k(A) & \longrightarrow & \pi_k(X) & \longrightarrow & \pi_k(X, A) \longrightarrow \cdots \\ & & \downarrow & & \downarrow & & \downarrow \\ \cdots & \longrightarrow & \tilde{H}_k(A) & \longrightarrow & \tilde{H}_k(X) & \longrightarrow & H_k(X, A) \longrightarrow \cdots \end{array}$$

Let X be an $(m - 1)$ -connected CW-complex. The Hurewicz theorem is proved as follows:

- (1) Use homotopy excision, and the homework problem on the split short exact sequence of the homotopy of a wedge, to show that the Hurewicz theorem holds for arbitrary wedges of spheres.
- (2) Use the argument for cellular approximation to show that X is homotopy equivalence to a CW complex Y with no cells below dimension m .
- (3) Apply Proposition 0.1 to perform induction on the skeletal filtration of Y , using homotopy excision and the case of spheres to handle the relative homotopy groups.