

HOMEWORK 9

DUE: WEDNESDAY, 4/19/06 (DUE TO HOLIDAY)

In the following problems, when I say “compute the Serre spectral sequence”, what I mean is

- (1) Identify E^2 .
- (2) Compute the differentials.
- (3) Analyze E^∞ and its relationship to the homology of the total space.

1. Let $S^1 \rightarrow S^3 \rightarrow S^2$ be the Hopf fibration. Compute the Serre spectral sequence for this fibration.

2. (Hatcher’s spectral sequence notes, Sec. 1.1, prob. 1) Let $\phi_n : S^k \rightarrow S^k$ be the degree n map for $n, k > 1$. Compute the homology of the homotopy fiber of ϕ_n .

3. (Hatcher’s spectral sequence notes, Sec. 1.1, prob. 2) Compute the Serre spectral sequence for the fibration

$$K(\mathbb{Z}/2, 1) \rightarrow K(\mathbb{Z}/8, 1) \rightarrow K(\mathbb{Z}/4, 1).$$

You may need to consult Hatcher’s spectral sequence notes, example 1.6, for guidance.

WARNING! This last problem actually turns about to be *much* more subtle than example 1.6.