

6 - Identification of E_2 , examples

Note Title

4/1/2010

Identification of E_2

$$\forall \text{ cells } D^s \xrightarrow{e_i^s} B$$

pick x_i, y_{i0k} , take a look! $D^s \times F \xrightarrow{x_i^s} E|_{e_i^s} \xrightarrow{y_{i0k}} D^s$

$$E'_{s,t} = H_{s+t}(E^s, E^{s-1})$$

→ $\hat{=} \tilde{H}_{s+t} \left(\coprod_i E|_{e_i^s}, \coprod_i E|_{\partial e_i^s} \right)$

compd
excision
argument

$$\cong \bigoplus_i \tilde{H}_{s+t}(F_{x_i} \times D^s, F_{x_i} \times \partial D^s)$$

$$\cong \bigoplus_i \tilde{H}_{s+t}(F_{x_i} \cup \sim D^s / \partial D^s)$$

$$\stackrel{\text{Kunnet}}{\cong} \bigoplus_i H_t(F_{x_i}) \otimes \tilde{H}_s(D^s / \partial D^s)$$

$$d_i: H_{s+t}(E^s, E^{s-1}) \rightarrow H_{s+t-1}(E^{s-1}) \rightarrow H_{s+t-1}(E^{s-1}, E^{s-2})$$

\mathbb{R} \mathbb{R}

$$\bigoplus_{\substack{i \in s\text{-cells} \\ (B)}} H_t(F_{x_i})$$

$$\bigoplus_{i \in (s-1\text{-cells})} H_t(F_{x_j})$$

$$\Rightarrow E_{s,t}^2 = H_s(B; H_t(F))$$

Examples!

$$E = F \times B$$

$$\downarrow$$

$$B$$

Spectral sequence collapses ($d_r = 0$ $r \geq 2$)

$$H_s(B; H_t(F)) \Rightarrow H_{s+t}(B \times F)$$

Universal coefficient

$$\text{Cor } H_n(B \times F) = \bigoplus_{s+t=n} H_s(B; H_t(F))$$

$$0 \rightarrow H_0(B) \otimes H_t(F) \rightarrow H_s(B; H_t(F)) \rightarrow \text{Tor}_1(H_{s-1}(B), H_t(F))$$

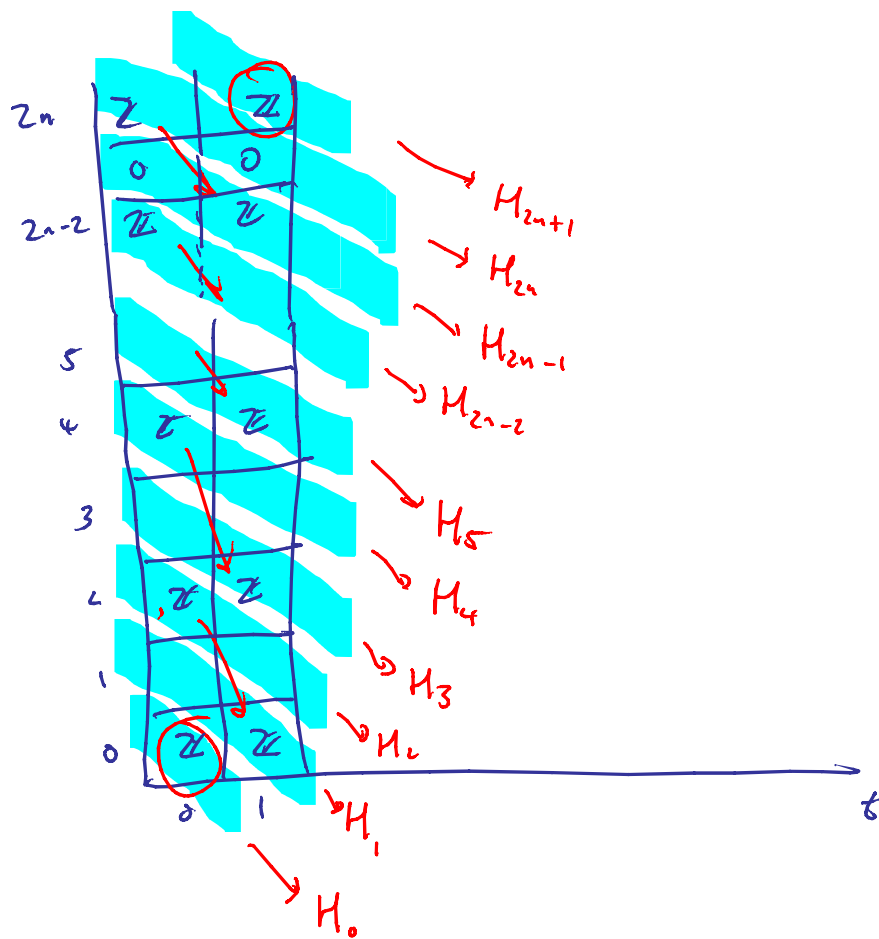
\downarrow
0

"Kunnet's theorem"

Example:

$$S^1 \rightarrow S^{2n+1} \rightarrow \mathbb{C}P^n$$

$$H_t(\mathbb{C}P^n; H_t(S^1)) \cong H_t(\mathbb{C}P^n) \otimes H_t(S^1)$$



Example

$$\mathbb{Z}/2 \rightarrow S^\infty \rightarrow \mathbb{R}P^\infty$$

$$H_0(F) = \mathbb{Z} \oplus \mathbb{Z} \cong \mathbb{Z}[\pi_1]$$



π_1 introduces free

$$H_s(\mathbb{R}P^\infty, H_+(F)) \begin{cases} \text{1 free spectral sequence} \\ t=0 \end{cases}$$

\Rightarrow collapses

$$H_s(\mathbb{R}P^\infty, H_0(F)) \cong H_s(S^\infty)$$

\cong

$$H_s(C_*^{\text{cell}}(\mathbb{R}P^\infty) \otimes_{\mathbb{Z}[\pi_1]} \mathbb{Z}[\pi_1])$$

\cong

$$H_s(C_*^{\text{cell}}(S^\infty))$$

Other examples!

$$H_+(S^{n+1})$$

✓
possible by
twisted
coeff's