Hw 1 solutions

Note Title 2/12/2009

2.1(3)

To show that the statement is false, we provide a counterexample.

Let
$$\{a_n\} = \{n^2\}_{n\geq 1} = \{1, 4, 9, \cdots\}$$
$$\{b_n\} = \left\{-\frac{1}{n}\right\}_{n\geq 1} = \left\{-1, -\frac{1}{2}, -\frac{1}{3}, \cdots\right\}$$

Then $\{a_n\}$ and $\{b_n\}$ are increasing,

but
$$\{a_n b_n\} = \left\{-\frac{n^2}{n}\right\} = \{-n\}$$
$$= \{-1, -2, -3, \cdots\}$$

is decreasing.

There are many correct answers to the second part. My suggestion is

Claim: If $\{a_n\}$ and $\{b_n\}$ are increasing sequences of positive numbers, then $\{a_n b_n\}$ is increasing.

(pf) By assumption:

$$a_n \le a_{n+1}$$
$$b_n \le b_{n+1}$$

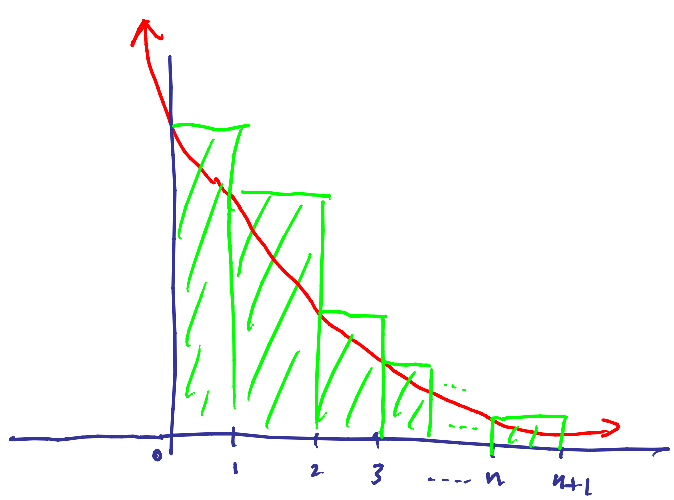
Since all are positive, we deduce

$$a_n b_n \le a_{n+1} b_{n+1}.$$

Thus, $\{a_n b_n\}$ is increasing. $\square$

2.3 (1)

(a) Compare with UPPER SUMS for the integral of $\frac{1}{3x+1}$



$$\text{Area (blocks)} = 1 + \tfrac{1}{4} + \tfrac{1}{7} + \cdots + \frac{1}{3n+1} = a_n$$
$$\text{\rotatebox{90}{$\le$}}$$
$$\int_0^{n+1} \frac{1}{3x+1}\,dx = \frac{\ln(3x+1)}{3}\Bigg|_{x=0}^{x=n+1} = \frac{\ln(3(n+1)+1)}{3}$$

Since $\frac{\ln(3(n+1)+1)}{3}$ increases without bound as n increases, so does a_n

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(b) <u>Strategy</u>:

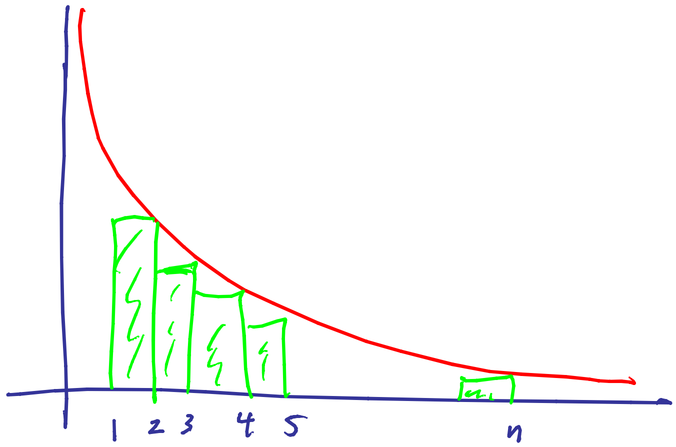
We will show that

$\{a_n - 1\}$ is bounded.

This suffices, because if M is an upper bound for $\{a_n - 1\}$, then $M+1$ is an upper bound for $\{a_n\}$.

$$a_n - 1 = \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \cdots + \frac{1}{n\sqrt{n}}$$

Compare w/ integral of $\frac{1}{x\sqrt{x}}$ using LOWER SUMS.



$$\text{Area}(\text{Blocks}) = a_n - 1$$

$$\leq$$

$$\int_1^\infty \frac{1}{x^{3/2}}\,dx = -2x^{-1/2}\Big|_1^\infty = 2$$

So 2 is an upper bound for $\{a_n - 1\}$

[<u>Note</u>: I used $a_n - 1$ instead of a_n because $\int_0^1 \frac{1}{x^{3/2}}\,dx = \infty$]