1. Bickel and Doksum, p. 73, no. 6.
   In more detail: suppose that the parameter ¸$\lambda > 0$ of a Poisson distribution has a $\Gamma(a, b)$ (Gamma distribution) prior distribution, where $a > 0$ and $b > 0$, so the prior density is
   $$\pi(\lambda) = \frac{b^a \lambda^{a-1} e^{-b\lambda}}{\Gamma(a)}.$$
   Suppose we observe X, a nonnegative integer having a Poisson(λ) distribution. Find the posterior distribution $\pi(\lambda|X)$ as in equation (1.2.8) of Bickel and Doksum. Show that the posterior distribution is again a $\Gamma(a', b')$ distribution and find a', b' in terms of a, b, and X.

2. Bickel and Doksum p. 78 Problem 8. Warning: the hint is not exactly correct. What is $E\left(\left[\frac{X_i-\mu}{\sigma}\right]^4\right)$ via integration by parts, and so what is $E((X_i - \mu)^4)$, actually?

3. Bickel and Doksum p. 78 Problem 9.

4. A statistic X is measured in a medical test. Suppose that for a certain disease, D, there are two possibilities. If the person being tested does not have D, the distribution of X is N(4, 1). If the person does have D, X has distribution N(7, 1).

(a). Suppose the test will be done for people in a “risk group” in which the prior probability of having D is 0.06. For any X, find the posterior probability given X that the patient has D.

(b) Suppose it costs \$50 each time X is measured for one patient. The physician has two available actions. One is to give a “negative” test result, deciding that the patient does not have D and doing no further tests or treatment for D. The other action is to give a “positive” result and then give a further test, based on a different statistic, costing \$1000, which will yield a correct result in essentially all cases. For some c, the test will be judged positive if X ¸c and negative otherwise. If a patient has D but the test is judged negative, assume a loss of \$1,000,000 (because the disease, left untreated, might become very serious). How should c be chosen to minimize the expected loss?

(c) In a general population where the prior probability of having D is 10^{-5}, is it cost-effective to do any such test procedure (for any c)? Hint: would it be, even if an initial \$50 test always gave the correct result?