Figure 1

The red lines are \( Z(P) \), where \( P \) is a minimal degree polynomial vanishing on the six points.
Figure 2
Stars

A grid with many 3-rich points

A grid with many 4-rich points

A grid with many 5-rich points.
(This picture is more complicated, so we draw it bigger.)
Black lines denote lines of $\mathcal{L}$. Dots denote 3-rich points of $\mathcal{L}$. Here we have $D = 2$ red lines dividing the plane into 4 cells.

The blue curve is $\gamma$. The black dots are points of $P_r(\mathcal{L})$. (The lines of $\mathcal{L}$ are not shown.)
Figure 5.

Original cutting

\[ \text{D red lines} \quad \Rightarrow \quad \text{Polynomial cutting} \]

\[ \text{degree } D \text{ alg. curve}, \quad Z(P), \ \text{drawn in red.} \]

In the picture \( D = 4 \). The curve on the right hand side is a union of two conics, so

\[ \text{Deg } P = 2 + 2 = 4. \]

Choosing \( D \) red lines involves \( 2D \) real parameters.

Choosing \( P \in \text{Poly}_D(\mathbb{R}^2) \) involves \( \text{Dim Poly}_D(\mathbb{R}^2) \sim D^2 \) real parameters.

In both cases, a line crosses the red walls at most \( D \) times and so enters at most \( D+1 \) cells.