LIST OF CLASSIC DIFFERENTIAL GEOMETRY PAPERS

Here is a list of classic papers in differential geometry that are suggestions for the seminar. I wrote a short description of each of them. I encourage you to work on something that looks interesting to you, and which looks challenging but not overwhelming. Depending on the size of the class, I think you'll get to present two or three things over the semester.

I organized this list by difficulty. Borrowing an idea from Canada/USA math camp, I described the difficulty of the papers in terms of chili peppers. If a paper is 'spicy' it may require more background or be long or complicated or just hard to read. For comparison, the paper we go through together in the first classes is about 2 chili peppers. If you don't have experience presenting papers in a seminar, you may want to start with a 1-2 chili paper, and you can decide how much to push yourself as the semester goes on...

The spicier papers will be hard to present in a single lecture. If you're intersted, you are welcome to study a paper as a team and give a sequence of lectures on it.

- Cheeger inequality (1 chili pepper) A lower bound for the smallest eigenvalue of the Laplacian. A fundamental estimate connecting isoperimetric inequalities and the spectrum of the Laplacian. Important in Riemannian geometry and also graph theory. A short lecture on this paper could perhaps be rounded out by introducing expanders.
- Cheeger and Gromoll. The splitting theorem for manifolds of nonnegative Ricci curvature. (1 chili pepper) A fundamental result on Ricci curvature. Influenced more recent work on Ricci curvature by Cheeger and Colding.
- Smale, Diffeomorphisms of the 2-sphere. (1 chili pepper) Smale finds the topology of the group of diffeomorphisms of S^2 by using the uniformization theorem.
- Milnor and Thurston, Characteristic numbers... A short paper with a striking connection between the topology and geometry of hyperbolic manifolds. Influenced Gromov's paper on volume and bounded cohomology below. (1-2 chili peppers)
- Milnor, On manifolds homeomorphic to the 7-sphere. (1-2 chili pepper) Surprising application of the signature theorem, showing that there are manifolds homeomorphic but not diffeomorphic to the 7-sphere.

- Bott, A topological obstruction to integrability (also called Bott vanishing theorem) (1-2 chili peppers) A short ingenious argument showing that certain homotopy classes of plane distributions do not contain any integrable distributions - i.e. do not contain any foliations.
- Cheeger and Gromoll. On the structure of complete manifolds of nonnegative curvature (1-2 chili peppers)
- Milnor, On the existence of a connection with curvature zero. (2 chili peppers) Surprisingly, there is an example of a 2-dimensional vector bundle on a closed surface (both oriented) which has a connection with curvature zero but has a non-zero Euler class. This is an interesting example about the relationship between curvature of bundles and characteristic classes. It contrasts with Chern's theorem, which says that if the connection preserves a metric on the bundle, then the Euler class can be represented by a polynomial in the curvature.
- Riemann-Roch theorem for Riemann surfaces. (2 chili peppers) This is a great theorem from the 19th century. In the 1950's, Hirzebruch generalized it to higher dimensions. His work influenced Atiyah and Singer, and the index theorem is closely related. The Riemann-Roch theorem is a special case of the index theorem - probably the first case of a theorem of this type, although I could be wrong. One reference is Fulton's book Algebraic Topology. Another reference - using more analysis and less topology - is Ahlfors's book on Riemann surfaces.
- The uniformization theorem. (2 chili peppers) Another great theorem in the history of mathematics. Ahlfors's book on Riemann surfaces is a classic reference.
- Gromov, Sign and geometric meaning of curvature, an expository essay (see Gromov's webpage, section 1, expository papers). This is an expository essay that looks quite readable to me on the geometric meaning of different inequalities about the Riemann curvature tensor, such as positive sectional curvature, negative sectional curvature, positive Ricci curvature, positive scalar curvature... It tries to give intuition.
- Bott periodicity theorem. See Milnor's book on Morse theory. (1-2 chili peppers) Bott applies Morse theory on the space of loops to find the homotopy groups of the unitary group. As an alternative, or as a first step, one could present Morse's results on the homology of the loop space of a sphere (also in Milnor's book).
- Li and Yau, On the parabolic kernel of the Schrodinger operator. Studies the heat kernel on Riemannian manifolds. How does the behavior of the heat kernel relate to the (Ricci) curvature. Also studies more general operators

and related variants of the curvature. Influenced Perelman's work on the Ricci flow mentioned below. (2 chili peppers)

- Smale's sphere eversion. A strange example in differential topology. One place to read about is the first chapter of the book Introduction to the h-principle, by Eliashberg and Misachev. (2 chili peppers)
- Gromov and Lawson, Spin and scalar curvature in the presence a fundamental group. (2-3 chili peppers). An application of the Atiyah-Singer index theorem.
- Schoen and Yau, Existence of incompressible minimal surfaces and the topology of three-dimensional manifolds with nonnegative scalar curvature. (2-3 chili peppers)
- Gromov, Curvature, diameter and Betti numbers. (3 chili peppers) Proves that a closed Riemannian manifold with non-negative sectional curvature has bounded Betti numbers. A fundamental result about topology and (sectional) curvature.
- Gromov, On the entropy of holomorphic maps. The entropy of a map from a manifold to itself is a measure of how strongly the space gets mixed up as you iterate the map. The paper proves that a holomorphic map from a Kahler manifold to itself has minimal possible entropy in its homotopy class. See also exposition by Cantat in L'enseignement math. and McMullen's paper Dynamics on blowups of the projective plane.
- Atiyah and Singer, On the index of elliptic operators (3 chili peppers) This is a classic paper with a great influence, including several papers in this list and many others. It's fundamental background for symplectic geometry and for gauge theory. There are several proofs with different strengths and weaknesses. I think it would be interesting to look at the first paper in the annals.
- Gromov, Volume and bounded cohomology (3 chili peppers) Builds on the work of Milnor-Thurston above to prove an interesting connection between Ricci curvature, topology, and volume.
- Perelman, The entropy formula for the Ricci flow and its geometric applications. (3-4 chili peppers) The whole proof of the Poincare conjecture is very long and complicated, but it would be plausible to present a piece of this paper. A plausible part of this paper to present is the monotonicity formula and a proof of no local collapsing. (There are two monotonicity formulas and two proofs of no local collapsing in the paper - the second is based on the Li-Yau paper above.) Although it takes a lot more work to prove the Poincare conjecture, this piece involves a real breakthrough. See also the notes by Kleiner-Lott and other sources.

- Yau, On the Ricci curvature of a compact Khler manifold and the complex Monge-Ampre equation. (3-4 chili peppers) Yau's proof of the Calabi conjecture is a great paper on geometric PDE. I think that a good place to read about this is Chap. 2 of Siu's book Lectures on Hermitian-Einstein Metrics for Stable Bundles and Kahler-Einstein Metrics.
- Thurston, The theory of foliations of codimension greater than one. (3-4 chili peppers) This paper complements Bott's vanishing theorem, mentioned above, proving that in many situations a plane distribution on a closed manifold can be homotoped to a foliation. For instance, this is possible if the normal bundle of the distribution is homotopically trivial. The flavor of the construction is similar to the Smale eversion above.
- Lusztig, Novikov's higher signature and families of elliptic operators. Application of the Atiyah-Singer index theorem to differential topology, studying the question when Pontryagin classes of a smooth manifold are homotopy invariant. This implies Novikov's theorem that (rational) Pontryagin classes are homeomorphism invariant. This paper involves the families version of the index theorem, so it requires some pretty strong index theory background. The families version of the index theorem is itself a great topic for a presentation. (3-4 chili peppers)