$\boxed{\text{Find the general solution to } \ddot{x} + 4x = t\sin 2t.}$

Our first step is to find a particular solution.

The input signal, $f(t) = t\sin 2t$, is neither a polynomial, exponential, the real or imaginary part of an exponential, nor partly exponential. So MUC, ERF, RERF, and ESL won't work right off the bat.

However, since $\sin 2t$ is the imaginary part of e^{2it}, we can still "go to complex numbers" and see if we have better luck there. We rewrite the input signal as $f(t) = \mathrm{Im}(te^{2it})$, and now want to solve the differential equation $\boxed{\ddot{z} + 4z = te^{2it}}$ with $\boxed{x = \mathrm{Im}(z)}$.

We can now use the ESL: $p(D)(e^{rt}u) = e^{rt}p(D + rI)(u)$. Since we want to eliminate the e^{2it} in the input signal of $\ddot{z} + 4z = te^{2it}$, we set $r = 2i$ and write $z = e^{2it}u$. We have the characteristic polynomial $p(s) = s^2 + 4$, so by the ESL, we get $\ddot{z} + 4z = p(D)(z) = p(D)(e^{2it}u) = e^{2it}p(D + 2iI)(u)$. This quantity is still equal to te^{2it}, as all we've done is replace z by $e^{2it}u$. Since $p(s) = s^2 + 4$, $p(D + 2iI) = (D + 2iI)^2 + 4I = D^2 + 4iD$, and $e^{2it}p(D + 2iI)(u) = e^{2it}(\ddot{u} + 4i\dot{u}) = te^{2it}$, so we can cancel the exponentials and end up with $\ddot{u} + 4i\dot{u} = t$.

The MUC doesn't work when the coefficient of u is zero (or, in the case of $\ddot{x} + b\dot{x} + kx$, when $k = 0$) so we can't use the MUC just yet. We first make another substitution $\boxed{v = \dot{u}}$ so our differential equation becomes $\dot{v} + 4iv = t$. Set $v = At + B$ and use MUC to find $A = -i/4$ and $B = 1/16$.

So $\boxed{v_p(t) = -it/4 + 1/16}$, and since $v = \dot{u}$, we have $\boxed{u_p(t) = -it^2/8 + t/16 + C}$. We know that $z_p(t) = e^{2it}u_p$, so $x_p = \mathrm{Im}(e^{2it}u_p)$. In other words,

$$x_p = \mathrm{Im}((\cos 2t + i\sin 2t)(-it^2/8 + t/16) = -\frac{1}{8}t^2\cos 2t + \frac{1}{16}t.$$

Combining with the homogeneous solutions of the original equation, we have, by superposition, $x = x_p + x_h$,

$$x(t) = -\frac{1}{8}t^2\cos 2t + \frac{1}{16}t\sin 2t + C_1\sin 2t + C_2\cos 2t.$$