I. Problem 3.1 on pg. 175
II. Problem 3.6 on pg. 175.
III. Problem 3.9 on pg. 176. You may assume that \( u \in C^3(\Omega) \cap C^1(\Omega) \).
IV. Problem 3.11 on pg. 176.
V. Let \( u(x) \) and \( f(x) \) be as in the previous problem. Show that \( u(x) \) solves Poisson’s equation \( \Delta u = -f \) (on \( \mathbb{R}^2 \)).
VI. Problem 3.21 on pg. 178.
VII. Let \( B_1(0) \) denote the solid unit ball in \( \mathbb{R}^n \), and let \( \partial B_1(0) \) denote its boundary. Let \( f(x) \) be smooth (i.e., infinitely differentiable) function on \( B_1(0) \), let \( g(\sigma) \) be a smooth function on \( \partial B_1(0) \), and let \( u(x) \) be a smooth solution to
\[
\begin{align*}
\Delta u(x) &= f(x), \quad x \in B_1(0), \\
u(\sigma) &= g(\sigma), \quad \sigma \in \partial B_1(0).
\end{align*}
\]
Show that there exists a constant \( C > 0 \) which does not depend on \( f \) or \( g \) such that
\[
|u(0)| \leq C \left( \max_{x \in B_1(0)} |f(x)| + \max_{\sigma \in \partial B_1(0)} |g(\sigma)| \right).
\]
If you prefer, you can supply a proof for the case \( n = 3 \) only (the remaining cases are similar).
Hint: Revisit the proof of the Mean value properties discussed in class.