Problem Set 4, **Due: at the start of class on 10-6-11**

I. Problem 3.1 on pg. 150.

II. Problem 3.2 on pg. 150.

III. Problem 3.3 on pg. 151. You may assume that $u \in C^3(\Omega) \cap C^1(\overline{\Omega})$.

IV. Problem 3.4 on pg. 151.

V. Problem 3.8 on pg. 152.

VI. Let $B_1(0)$ denote the solid unit ball in $\mathbb{R}^n$, and let $\partial B_1(0)$ denote its boundary. Let $f(x)$ be smooth (i.e., infinitely differentiable) function on $B_1(0)$, let $g(\sigma)$ be a smooth function on $\partial B_1(0)$, and let $u(x)$ be a smooth solution to

$$\Delta u(x) = f(x), \quad x \in B_1(0),$$

$$u(\sigma) = g(\sigma), \quad \sigma \in \partial B_1(0).$$

Show that there exists a constant $C > 0$ which does not depend on $f$ or $g$ such that

$$\max_{x \in B_1(0)} |u(x)| \leq C \left( \max_{x \in B_1(0)} |f(x)| + \max_{\sigma \in \partial B_1(0)} |g(\sigma)| \right).$$

If you prefer, you can supply a proof for the case $n = 3$ only (the remaining cases are similar).

**Hint:** Revisit the proof of the Mean value properties discussed in class.