Ex. Police are 50 ft. from the side of the road. Their radar sees your car approaching at 80 ft/sec when your car is 50 ft. from the radar gun. The speed limit is 65 miles/hr. $\approx 95$ ft/sec.

Note that we have given names to the variables.

It is very important to figure out which variables are changing and which are constant. In this problem, $x$ and $D$ are changing as $t$ varies, where $t =$ time.

Relationships between variables:

By the Pythagorean Thm, $x^2 + 30^2 = D^2$ (*).

When $D = 50$, we solve for $x$: 

\[ x = 40 \]

We then implicitly differentiate the eqn (*), with respect to $t$:

\[ \frac{d}{dt} (x^2 + 30^2) = \frac{d}{dt} (D^2) \]

Using the chain rule, we have:

\[ 2x \frac{dx}{dt} = 2D \frac{dD}{dt} \]

Thus, $x' = \frac{D}{x} D'$, where $\frac{d}{dt}$. 

\[ L10.1 \]
• Now we simply plug in \( x = 40 \), \( D = 50 \), \( \dot{D} = -80 \)
to deduce:
\[
x' = \frac{50}{40} (-80) = -100 \text{ ft/sec} \Rightarrow \text{You are speeding.}
\]

• Note that \( x' < 0 \), which makes sense since \( x \) is decreasing.

• You could also have solved this problem by first
solving for \( D \) in terms of \( x \):
\[
D = \sqrt{30^2 + x^2}.
\]

You could then differentiate this equation with respect
to \( t \) and use the chain rule to deduce that
\[
\frac{dD}{dt} = \frac{1}{2} (30^2 + x^2)^{-\frac{1}{2}} \cdot 2x \cdot \frac{dx}{dt}.
\]

• You could then plug in as above to find \( \frac{dx}{dt} \).

• However, the algebra is often more complicated when
you try to explicitly solve for the one variable in terms
of the other.
- Overall Strategy

1. Draw picture
2. Set up variables + equations
3. Take derivatives
4. Plug in the values after taking the derivatives

Ex:

Consider a conical tank with \( r_{\text{tank}} = 4, \ h_{\text{tank}} = 10 \). Suppose it is being filled with water at a rate of 2 ft\(^3\)/min. How fast is the water rising when it is 5 ft. high?

- The volume of water in the tank is \( V = \frac{1}{3} \pi r^2 h \).
- Using similar triangles (side view), we have: \( \frac{r}{h} = \frac{4}{10} \)
  \( \Rightarrow r = \frac{2}{5} h \)
• Plugging into $V$, we have

$$\frac{dV}{dt} = \frac{4}{25} \pi \cdot h^2 \cdot \frac{dh}{dt}$$

• Differentiating with respect to $t$ and using the chain rule, we have:

$$\frac{dV}{dt} = \frac{4}{25} \pi \cdot h^2 \cdot \frac{dh}{dt}$$

• Plugging in $\frac{dV}{dt} = 2$ and $h = 5$, we have

$$2 = \left(\frac{4}{25} \pi\right) \cdot 5^2 \cdot \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{1}{2 \pi}.$$