a. Linear approximations
   (a) \( f(x) = f(x_0) + f'(x_0)(x - x_0) + O((x - x_0)^2) \) for \( x \) near \( x_0 \)

b. Quadratic approximations
   (a) \( f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 + O((x - x_0)^3) \) for \( x \) near \( x_0 \)

c. Graphing
   (a) Zeros
      (i) Are points \( x \) where \( f(x) = 0 \)
   (b) Critical points
      (i) Are points \( x \) where \( f'(x) = 0 \)
      (ii) Help identify local mins and maxes
      (iii) Not all critical points correspond to a min or a max
   (c) Concavity
      (i) \( f''(x) > 0 \implies \) concave up
      (ii) \( f''(x) < 0 \implies \) concave down
      (iii) Inflection points
         (A) Are points where \( f''(x) = 0 \)
         (B) Help identify points where the concavity changes from up to down or vice versa
         (C) Not all inflection points correspond to changes in concavity
   (d) Second derivative test
      (i) If \( x \) is a critical point and \( f''(x) > 0 \), then \( x \) is a local min
      (ii) If \( x \) is a critical point and \( f''(x) < 0 \), then \( x \) is a local max
      (iii) If \( x \) is a critical point and \( f''(x) = 0 \), then more information is needed to determine whether \( x \) is a local min or a local max (or neither)
   (e) Watch out for points of discontinuity or non-differentiability
   (f) Make sure to indicate limiting behavior as \( x \to \pm \infty \) or \( x \to \) a point of discontinuity

d. Max-min problems
   (a) If \( f(x) \) is continuous on \([a, b]\), then the min and max occur at an endpoint, a critical point, or a “bad point” (i.e., a point of non-differentiability)

e. Related rates of change
   (a) The key to these problems is to differentiate the variable relationships and to apply the chain rule.
Midterm 2 - Review Sheet

f. Newton’s method
   (a) \( x_{k+1} = x_k - f(x_k)/f'(x_k) \)

g. Mean value theorem
   (a) \( f(b) - f(a) = f'(c)(b - a) \) for some \( c \) in between \( a \) and \( b \)
   (b) \( \frac{f(b)-f(a)}{b-a} = f'(c) \); average rate of change of \( f \) is equal to the instantaneous rate of change of \( f \) at some point in between
   (c) \( f' \geq 0 \) implies that \( f \) is increasing
   (d) \( f' \leq 0 \) implies that \( f \) is decreasing
   (e) If \( f'(x) = 0 \) for all \( x \), then \( f \) is constant-valued

h. Differentials
   (a) Are essentially equivalent to linear approximation
   (b) \( \Delta y \approx dy = f'(x)dx \)

i. Antiderivatives
   (a) \( F' = f \) means that \( F \) is an antiderivative of \( f \)
   (b) If \( F' = f \) and \( G' = f \), then \( F(x) = G(x) + c \) for some constant \( c \)
   (c) Substitution method
      (i) \( \int f(u(x))u'(x)dx = \int f(u)du \)
   (d) Basic examples (this is only a partial list)
      (i) \( \int \sin x \, dx = -\cos x + c \)
      (ii) \( \int \cos x \, dx = \sin x + c \)
      (iii) \( \int x^n \, dx = \frac{1}{n+1}x^{n+1} + c, \, n \neq -1 \)
      (iv) \( \int x^{-1} \, dx = \ln |x| + c \)
      (v) \( \int (1 + x^2)^{-1} \, dx = \arctan x + c \)
      (vi) \( \int (1 - x^2)^{-1/2} \, dx = \arcsin x + c_1 = -\arccos x + c_2 \)
      (vii) \( \int e^x \, dx = e^x + c \)