18.103 Fall 2013

Problem Set 6 (due 10am Fri, Oct 18)

Exercise 1. Let $\mathcal{M}_n = \mathcal{M}(\mathbb{R}^n)$, the measurable subsets of $\mathbb{R}^n$. Show that $\mathcal{M}_1 \times \mathcal{M}_1 \neq \mathcal{M}_2$ by considering $E \times \{0\}$ with $E \subset [0,1]$ such that $E \notin \mathcal{M}_1$.

Exercise 2. Show that if $f : \mathbb{R}^2 \to \mathbb{R}$ is measurable with respect to $\mathcal{M}(\mathbb{R}^2)$, then there is a Borel function g such that $f(x) = g(x)$ for almost every $x \in \mathbb{R}^2$. (Hint: Start with the case $f = 1_E$ and use a similar approach to §2.2 Theorem 6, page 62.)

Exercise 1 gives an example of a function, namely, $1_{E\times\{0\}}$ that is $\mathcal{M}(\mathbb{R}^2)$ measurable but whose slices need not be $\mathcal{M}(\mathbb{R})$ measurable. What Exercise 2 shows is that by modifying an $\mathcal{M}_2$ measurable function on a set of measure zero, we can turn it into a function to which Fubini's theorem as stated in the text applies.

AG §2.1, p 60: 11b (Read 11a to learn the terminology of pointwise convergence and convergence in measure. We already did 11a as the extra exercise on PS2. This is the statement that the conclusion of the strong law implies the conclusion of the weak law of large numbers.)

AG §3.1, pp 122–124: 1, 2, 3, 7, 8

Warning: The sentence at the top of page 122 concerning a "corollary to the Lebesgue dominated convergence theorem" is misleading. The dominated convergence theorem is not needed in the proof of the completeness of the Lebesgue spaces L^p. The part of the corollary that is used does not employ the dominated convergence theorem. As explained in lecture, we want to *conclude* that there is pointwise convergence, whereas the dominated convergence theorem has pointwise convergence as a hypothesis.

AG §3.2, pp 128–129: 1, 2, 4

AG §3.3, pp 134–137: 6 (You may use the Pythagorean theorem, Theorem 2 from §3.3 or derive what you need from direct computation as we have already done a few times for Rademacher functions. Otherwise, don't use the theorems in §3.3 for this; do it with the theorems from §3.2. In other words, you don't need §3.3 to do this exercise.)