

Dark Matter and Dark Energy

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Abstract

We suggest that a candidate for dark matter is a meson with spin one the existence of which is dictated by local scale invariance proposed by Herman Weyl..

Dark Matter and Scale Invariance

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We suggest that the dark matter is a boson with spin unity. Indeed, it is the vector meson S first proposed by Weyl on the basis of scale invariance. We point out that, in our theory of local scale invariance, there is an inherent mechanism of mass generation which produces particle masses. As a result, scale invariance is spontaneously broken and our theory does not have the malaise of mass anomalies which plagues theories of global mass invariance. We also show that, since the S meson does not couple with the fermions, this theory is unitary. Besides producing particle masses, the mass generation mechanism also produces the observed dark energy.

Since the S meson is a boson, it may form a condensate which is translucent to light. The gravitational field generated by this condensate bends the light beam that passes through it. Therefore, a translucent condensate, if it exists, acts like a lens to light.

As colliders are being built with the creation of dark matters among the stated goals, we wish to point out that one of the possible candidates of dark

matter is the vector meson S first conceived by Weyl on the basis of scale invariance^{1,2}.

Weyl made the mistake of identifying S with the photon. Also, Weyl's work is incomplete, as he lacked at his time the framework of quantum electrodynamics and the theory of general relativity on which our theory of scale invariance is based. Our formulation of scale invariance yields results³ different from those of Weyl's. For example, we find that S does not couple with the electron. It is well-known that Einstein argued that electrons moving in a background of the S field would produce unobserved broadening of the atomic spectral lines. That the electron does not interact with S overrides this objection of Einstein. Indeed, S is a kind of dark matter coupling with neither quarks and leptons, nor any of the gauge mesons. Indeed, in our theory, the only interactions S has are those with the graviton and scalar mesons, the only candidate of which at the moment is the Higgs meson.

Let $g_{\mu\nu}$ be the metric tensor. Then the distance between two neighboring space-time points is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu.$$

Let us change the scale of distance globally, e.g., changing the unit of length from the centimeter to the inch for every point in space. With this change, the distance remains the same, but is measured in a different unit. This can be done by changing the metric tensor from $g_{\mu\nu}$ to $g'_{\mu\nu}$, where

$$g'_{\mu\nu} = \Lambda^2 g_{\mu\nu}, \tag{1}$$

with Λ a constant. Then we have

$$ds'^2 = \Lambda^2 ds^2,$$

where

$$ds'^2 \equiv g'_{\mu\nu} dx^\mu dx^\nu.$$

Thus, with the transformation (1), the numerical value of the distance between two given points is changed by a constant multiple. Since $g^{\mu\nu}$ is the inverse of $g_{\mu\nu}$, we have from (1) that

$$g'^{\mu\nu} = \Lambda^{-2} g^{\mu\nu}. \tag{2}$$

We also have from (1) that

$$(-\det g')^{1/2} = \Lambda^4 (-\det g)^{1/2}, \tag{3}$$

where $\det g$ is the determinant of the matrix $g_{\mu\nu}$.

The tetra ε_μ^a satisfies

$$\eta_{ab} \varepsilon_\mu^a \varepsilon_\nu^b = g_{\mu\nu},$$

where η_{ab} is the metric tensor in the Minkowski space. Thus the tetrad

transforms like

$$\varepsilon'^a_\mu = \Lambda \varepsilon^a_\mu. \quad (4)$$

From (2) and (4), we have

$$\varepsilon'^{a\mu} = \Lambda^{-1} \varepsilon^{a\mu}, \quad (5)$$

where

$$\varepsilon^{a\mu} = g^{\mu\nu} \varepsilon^a_\nu.$$

It is to be noted that many field theories found useful today are globally scale invariant. To see this, we first mention that the action is given by

$$\int d^4x (-\det g)^{1/2} L,$$

where L is the Lagrangian density. By (3), the action is invariant under global scale transformations provided that

$$L' = \Lambda^{-4} L. \quad (6)$$

Consider the Lagrangian of a massless scalar field ϕ with a ϕ^4 coupling which is given by

$$\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \lambda \phi^4, \quad (7)$$

where λ is the quartic coupling constant. By (2), the Lagrangian of (7) satisfies (6) provided that

$$\phi' = \Lambda^{-1} \phi. \quad (8)$$

Next consider the Lagrangian density of the electromagnetic field A_μ

$$-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}, \quad (9)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

By (2), this Lagrangian density satisfies (6) provided that

$$A'_\mu = A_\mu. \quad (10)$$

The Yang-Mills theory is also invariant under global scale transformations. This is because

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc} W^b_\mu W^c_\nu$$

is invariant under global scale transformations provided that

$$W'^a_\mu = W^a_\mu, \quad (11)$$

where W^a_μ is the Yang-Mills gauge meson, g is a dimensionless coupling constant, and f^{abc} is the structure constant of the gauge group.

The Lagrangian for the electron field Ψ coupling with the electromagnetic field and the gravitational field is

$$\bar{\Psi} i \gamma^c \varepsilon^\mu_c \left[\partial_\mu + i e A_\mu - \frac{1}{2} \sigma_{ab} \varepsilon^{b\nu} (\partial_\mu \varepsilon^a_\nu - \Gamma^\rho_{\mu\nu} \varepsilon^a_\rho) \right] \Psi, \quad (12)$$

where

$$\Gamma_{\mu\nu}^\rho = \frac{g^{\rho\sigma}}{2}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}),$$

and

$$\sigma^{ab} = \frac{\gamma^a \gamma^b - \gamma^b \gamma^a}{4}.$$

It is straightforward to find that this Lagrangian density satisfies (6) provided that

$$\Psi' = \Lambda^{-3/2} \Psi. \quad (13)$$

Let us see what happens if we demand the theory be scale invariant locally, i.e., when Λ is a function of time and space. With the usual arguments we use to deduce the existence of gauge fields, we find that all the theories above become locally scale invariant provided that there exists a vector meson S and we make in these Lagrangian the replacements

$$\begin{aligned} \partial_\mu g_{\nu\rho} &\rightarrow (\partial_\mu + 2fS_\mu)g_{\nu\rho}, \quad \partial_\mu g^{\nu\rho} \rightarrow (\partial_\mu - 2fS_\mu)g^{\nu\rho}, \\ \partial_\mu \varepsilon_\nu^a &\rightarrow (\partial_\mu + fS_\mu)\varepsilon_\nu^a, \quad \partial_\mu \varepsilon_a^\nu \rightarrow (\partial_\mu - fS_\mu)\varepsilon_a^\nu, \\ \partial_\mu \phi &\rightarrow (\partial_\mu - fS_\mu)\phi, \quad \partial_\mu \Psi \rightarrow (\partial_\mu - \frac{3}{2}fS_\mu)\Psi. \end{aligned} \quad (14)$$

In the above, f is a coupling constant which is the counterpart of e in quantum electrodynamics.

As an example, with the replacement (14), the Lagrangian of (7) becomes

$$\frac{1}{2}g^{\mu\nu}(\partial_\mu - fS_\mu)\phi(\partial_\nu - fS_\nu)\phi - \lambda\phi^4. \quad (15)$$

The Lagrangian density above satisfies (6) with

$$\phi' = \Lambda^{-1}\phi, \quad g'^{\mu\nu} = \Lambda^{-2}g^{\mu\nu},$$

and

$$S'_\mu = S_\mu - \frac{1}{f}\partial_\mu \ln \Lambda, \quad (16)$$

where Λ is a function of space and time.

By (10) and (11), there is no need to alter the Lagrangian for a gauge meson. Thus gauge mesons do not couple with S .

Neither does a fermion. The proof is straightforward but requires a little algebra so we will give the details below. We note that, as we make the replacement (14), the additional terms generated from

$$\begin{aligned} &-\frac{\gamma^c}{2}\sigma_{ab}\varepsilon^{b\nu}\varepsilon_c^\mu(\partial_\mu \varepsilon_\nu^a - \Gamma_{\mu\nu}^\rho \varepsilon_\rho^a) \\ \text{are} \\ &-\frac{\gamma^c}{2}\sigma_{ab}\varepsilon^{b\nu}\varepsilon_c^\mu f(-S_\nu \varepsilon_\mu^a + g^{\rho\sigma} S_\sigma g_{\mu\nu} \varepsilon_\rho^a). \end{aligned}$$

The expression above can be simplified into

$$\frac{\gamma^c}{2}f(\sigma_{cb}\varepsilon^{b\nu}S_\nu - \sigma_{ac}\varepsilon^{a\sigma}S_\sigma) = f\gamma^c\sigma_{cb}\varepsilon^{b\nu}S_\nu.$$

(17)

Now

$$\gamma^c \sigma_{cb} = \gamma^c \frac{\gamma_c \gamma_b - \gamma_b \gamma_c}{4} = \frac{3\gamma_b}{2}.$$

Thus the expression in (17) is equal to

$$\frac{3}{2} f \gamma^b \varepsilon_b^\nu S_\nu. \quad (18)$$

Also, the term generated from $\partial_\mu \Psi$ is

$$-\frac{3}{2} f S_\mu \Psi. \quad (19)$$

Therefore, as we make the replacement (14), the terms generated from the Lagrangian (12) completely cancel one another. Thus we conclude that the fermion does not couple with S .

While there are no mass terms in the Lagrangian of our theory, there is an inherent mass generation mechanism in a locally scale invariant theory. As we know, there are a number of mechanisms which spontaneously generate masses⁴, a notable example being the Higgs model⁵. The mass generation mechanism in a locally scale invariant theory is distinct from any of the mechanisms mentioned above. For example, the potential of the scalar field in a scale invariant theory cannot be a Higgs potential, having only a quartic term ϕ^4 .

Consider as a simple model in which there is a real scalar meson ϕ , the S meson, and a gravitational field. The Lagrangian in this model³ is given by the sum of (15) and

$$-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} H_{\mu\nu} H_{\rho\sigma} + \frac{1}{2} \beta \phi^2 \tilde{R}, \quad (20)$$

where

$$H_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu,$$

and $\tilde{R}$ is equal to the scalar curvature R with the derivative replaced by the scale covariant derivative defined in (14). Also, β is a dimensionless constant. This theory is scale invariant, and the magnitude of ϕ can be altered by making a scale change. As a side remark, we note that if ϕ is equal to zero at a certain space-time point, it remains to be zero at this point under any scale transformation. This is to say that the zeroes of ϕ can never be made to disappear with a scale transformation. Thus two sets of ϕ solutions having different zeroes are not equivalent.

Consider the case in which ϕ is positive definite at all space-time point. Then we may make, for example,

$$\phi(\vec{x}, t) = 1$$

for all space-time points. We may also make ϕ to equal to, say, 1000 at all points; but such a solution is equivalent to $\phi = 1$. This is because $\phi = 1\text{Gev}$, say, is the same as $\phi = 1000\text{Mev}$. Clearly, the numerical value of ϕ is not meaningful until we specify the unit we express it in, and we have the freedom to choose this unit. Once the unit is specified, the scale of the theory is set. We shall make ϕ be $v/\sqrt{2}$ at all space-time points. Then the Lagrangian (15) becomes

$$-\frac{f^2 v^2}{4} S^2 - \lambda v^4/4. \quad (21)$$

The first term of (21) is a mass term of the S meson. The second term in (21) gives an energy density $\lambda v^4/4$, which is the energy density of the universe in the absence of any particles. Therefore, λv^4 is the cosmological constant, or the density of the dark energy^{3,7} the existence of which has been deduced in astrophysical observations.

We mention that, in addition to having the physical consequences of giving rise to particle masses and the dark energy, this scheme of symmetry breaking also has the conceptual appeal that the negative quadratic term in the Higgs potential is not required.

There is also a mass term for S coming from the second term of (20). We have, with ϕ replaced by $v/\sqrt{2}$,

$$\frac{1}{2} \beta \phi^2 \tilde{R} = \frac{1}{16\pi G} (R + 6f^2 S^2 + 6f D_\mu S^\mu), \quad (22)$$

where

$$G = (4\pi\beta v^2)^{-1}$$

is the gravitational constant and

$$D_\mu S^\mu = \partial_\mu S^\mu + \Gamma_{\mu\nu}^\mu S^\nu.$$

The last term in (22) is a surface term and can be ignored. This is because $(-\det g)^{1/2} D_\mu S^\mu = \partial_\mu [(-\det g)^{1/2} S^\mu]$.

Thus the terms in the Lagrangian involving the S field is simply

$$-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} H_{\mu\nu} H_{\rho\sigma} + \frac{1}{2} M_s^2 g^{\mu\nu} S_\mu S_\nu,$$

where M_s is the S mass. This shows that the S meson couples with the gravitational field through its energy-momentum tensor. Also, the equation of motion for S is

$$g^{\mu\rho} D_\mu H_{\rho\nu} = -M_s^2 S_\nu.$$

The mass term in (21) is much smaller than the mass term given in (22). Thus M_s is approximately given by

$$M_s \simeq f \sqrt{\frac{3}{4\pi G}}. \quad (23)$$

Let us now see what happens as we impose scale invariance on the standard model. We add the S field and the gravitational field to the theory in the way described previously, and replace the Higgs potential of the iso-doublet of scalar mesons in the standard model by a quartic potential. By making a rotation in the iso-space, we may make the iso-doublet of scalar meson to have a real and positive lower component with no upper component. Assuming that this lower component does not vanish at any space-time point, we may make a scale change to set it to equal to $v/\sqrt{2}$ at all points. Thus $W^\pm$, Z , quarks and leptons get masses in exactly the same way as in the standard model. The only difference is that, with all four components of the scalar iso-doublet eaten up by $W^\pm$, Z and S , there exists no physical Higgs meson.

This theory, with gravity incorporated, is not renormalizable. Even if we ignore the gravitational interaction in this theory, it still appears to be not renormalizable. This is because a massive Yang-Mills theory is known to be not renormalizable⁶.

The locally scale invariant theory proposed here, with scale invariance broken spontaneously, does not have mass anomalies which plague theories of global scale invariance.

That the vector meson S does not couple to the electron avoids a difficulty which is different from the one raised by Einstein. This is because the Lagrangian (12) would be complex if there were a term $f S_\mu \gamma^\mu$ inside the bracket of (12). On the other hand, the Lagrangian (15) is real, hence the corresponding Hamiltonian is real. This Hamiltonian becomes a hermitian operator after quantization, and our theory is unitary.

Nuclei are formed as protons and neutrons attract one another with strong interactions. Solids or liquids are formed as electrons and nuclei have the electromagnetic interaction. Since the S mesons have but gravitational interactions, they cannot form the kinds of matter we see. Thus the S particles mostly move freely in space, with nothing to reveal their existence other than through the gravitational field they generate. On the other hand, unlike electrons and nucleons, S is a boson and does not satisfy the exclusion principle. As a result, there is no limit to how many S particles may occupy the same spatial point at the same time. Therefore, when the density of such particles become sufficiently high, they can be bound together by gravitational forces. Thus it may be possible for them to form a condensation the behaviors of which are different from those of ordinary matters. In particular, they are translucent to light and can be heavy. The gravitational field generated from

such a condensate bends the light that passes through it. Thus a translucent condensate, if it exists, acts like a lens that bends light.

It is interesting to ponder if there are ways to observe the S particles if they exist in nature.

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