ON THE CONSTRUCTION FK

John Milnor

(lecture notes from Princeton University, 1956)

1. Introduction

The reduced product construction of Ioan James [5] assigns to each CW-complex a new CW-complex having the same homotopy type as the loops in the suspension of the original. This paper will describe an analogous construction proceeding from the category of semi-simplicial complexes to the category of group complexes. The properties of this construction FK are studied in §2.

A theorem of Peter Hilton [4] asserts that the space of loops in a union $S_1 \vee \ldots \vee S_r$ of spheres splits into an infinite direct product of loops spaces of spheres. In §3 the construction of FK is applied to prove a generalization (Theorem 4) of Hilton's theorem in which the spheres may be replaced by the suspensions of arbitrary connected (semi-simplicial) complexes.

The author is indebted to many helpful discussions with John Moore.

2. The construction

It will be understood that with every semi-simplicial complex there is to be associated a specified base point.
Let $K$ be a semi-simplicial complex with base point $b_0$. Denote $S_0 b_0$ by $b_n$. Let $F_K^n$ denote the free group generated by the elements of $K_n$ with the single relation $b_n = 1$. Let the face and degeneracy operations $\partial_i, s_i$ in $F_K = U F_K^n$ be the unique homomorphisms which carry the generators $k_n$ into $\partial_i k_n, s_i k_n$ respectively. Thus each complex $K$ determines a group complex $F_K$.

It will be shown that $F_K$ is a loop space for $E_K$, the suspension of $K$. (Definitions will be given presently.)

Alternatively let $F^+ K_n \subseteq F_K^n$ be the free monoid (= associative semi-group with unit) generated by $K_n$, with the same relation $b_n = 1$. Then the monoid complex $F^+ K$ is also a loop space for $E_K$. This construction is the direct generalization of James' construction. (See Lemma 4.)

The suspension $E_K$ of the semi-simplicial complex $K$ is defined as follows. For each simplex $k_n$ other than $b_n$ of $K$ there is to be a sequence $(E_k^n), (s_0 E_k^n), (s_0^2 E_k^n), \ldots$ of simplexes of $E_K$ having dimensions $n + 1, n + 2, \ldots$. In addition there is to be a base point $(b_0)$ and its degeneracies $(b_n)$. The symbols $(s_0^i Eb_n)$ will be identified with $(b_{n+i+1})$. The face and degeneracy operations in $E_K$ are given by

$$\begin{align*}
\partial_j (E_k^n) &= (E \partial_{j-1}^n k_n) & (j > 0, \ n > 0) \\
s_j (E_k^n) &= (E s_{j-1}^n k_n) & (j > 0) \\
\partial_0 (E_k^n) &= (b_0) & \partial_1 (E_k^n) = (b_n) \\
s_0 (E_k^n) &= (s_0 E_k^n).
\end{align*}$$

The face and degeneracy operations on the remaining simplexes

\[120\]
\[(s_0^i E_k)_n = s_0^i (E_k)_n \] are now determined by the identities

\[
\partial_j s_0^i = \begin{cases} 
  s_0^i \partial_j \bar{l} & (j > i) \\
  s_0^{i-1} & (j \leq i) \neq 0
\end{cases}
\]

\[
s_0^i s_j^i = \begin{cases} 
  s_0^i s_{j-1} & (j > i) \\
  s_0^{i+1} & (j \leq i)
\end{cases}
\]

It is not hard to show that this defines a semi-simplicial complex. The following lemma will justify calling it the suspension of $K$. Recall that the suspension of a topological space $A$ with base point $a_0$ is the identification space of $A \times I$ obtaining by collapsing $(A \times \{1\}) \cup (a_0 \times I)$ to a point.

**Lemma 1.** The geometric realization $|E_k|$ is canonically homeomorphic to the suspension of $|K|$.

(For the definition of realization see [6]. In fact the required homeomorphism is obtained by mapping the point $(|k_n^1, \delta_n^1|, 1-t)$ of the suspension of $|K|$, where $\delta_n^1$ has barycentric coordinates $(t_0^1, \ldots, t_n^1)$ into the point $(|E_k^1, \delta_{n+1}^1| \in |E_k|$, where $\delta_{n+1}^1$ has barycentric coordinates $(1-t, t_0^1, \ldots, t_n^1)$).

Next the space of loops on a semi-simplicial complex $K$ will be discussed. If $K$ satisfies the Kan extension condition then $\Omega K$ can be defined as in [7]. This definition has two disadvantages:

(1) Many interesting complexes do not satisfy the extension condition. In particular $E_k$ does not.
(2) There is no natural way (and in some cases \(^\dagger\) no possible way) of defining a group structure in \(\Omega K\).

The following will be more convenient. A group complex \(G\), or more generally a monoid complex, will be called a loop space for \(K\) if there exists a (semi-simplicial) principal bundle with base space \(K\), fibre \(G\), and with contractible total space \(T\).

(By a principal bundle is meant a projection \(p\) of \(T\) onto \(K\) together with a left translation \(G \times T \to T\) satisfying

\[(g_n \cdot g'_n) \cdot t_n = g_n \cdot (g'_n \cdot t'_n)\]

where \(g_n \cdot t_n = t'_n\) if and only if \(g_n = 1_n\); and where \(g_n \cdot t_n = t'_n\) for some \(g_n\) if and only if \(p(t_n) = p(t'_n)\). A complex is called contractible if its geometric realization is contractible. This is equivalent to requiring that the integral homology groups and the fundamental group be trivial.)

The existence of such a loop space for any connected complex \(K\) has been shown in recent work of Kan, which generalizes the present paper. The following Lemma is given to help justify the definition.

**Lemma 2.** If \(K\) satisfies the extension condition, and the group complex \(G\) is a loop space for \(K\), then there is a homotopy equivalence \(\Omega K \to G\).

\(^\dagger\) Let \(K\) be the minimal complex of the \(n\)-sphere \(n \geq 2\). Then it can be shown that there is no group complex structure in \(\Omega K\) having the correct Pontrjagin ring.
(10)

The proof is based on the following easily proven fact (compare [7] p. 2-10): Every principal bundle can be given the structure of a twisted cartesian product. That is one can find a one-one function

$$\eta: G \times K \to T$$

satisfying $\partial_i \eta = \eta \partial_i$ for $i > 0$ and $s_i \eta = \eta s_i$ for all $i$, where $\partial_0 \eta$ is given by an expression of the form

$$\partial_0 \eta(g_k, k_n) = \eta(\partial_0 g_n, (\tau_k, \partial_0 k_n)).$$

(For this assertion the fibre must be a monoid complex satisfying the extension condition.) Thus the bundle is completely described by $G$ and $K$ together with the 'twisting function' $\tau: K_n \to G_{n-1}$, where $\tau$ satisfies the identities

$$s_i \tau = \tau s_{i+1} \quad (i \geq 0), \quad \partial_1 \tau = \tau \partial_{i+1} \quad i \geq 1,$$

$$\tau s_0 k_n = 1_n, \quad (\partial_0 \tau_k, (\tau \partial_0 k_n) = \tau \partial_1 k_n.$$ 

Now a map $\overline{\tau}: \Omega K_{n-1} \to G_{n-1}$ is defined by $\overline{\tau}(k_n) = \tau(k_n)$. From the definition of $\Omega K$ and the above identities it follows that $\overline{\tau}$ is a map. From the homotopy sequence of the bundle it is easily verified that $\overline{\tau}$ induces isomorphisms of the homotopy groups, which proves Lemma 2.

To define a principal bundle with fibre $FK$ and base space $EK$ it is sufficient to define twisting functions $\tau: EK_{n+1} \to FK_n$. These will be given by

$$\tau(Ek_n) = k_n, \quad \tau(s^i_0 Ek_{n-1}) = 1_n \quad (i > 0).$$
Theorem 1. FK is a loop space for EK. In fact the twisted cartesian product \( \{FK, EK, \tau\} \) has a contractible total space.

It is easy to verify that \( \tau \) satisfies the conditions for a twisting function. Hence we have defined a twisted cartesian product, and therefore a principal bundle. Let \( T \) denote its total space. Note that \( T \) may be identified with \( FK \times EK \) except that \( \partial_0 \) is given by

\[
\partial_0(g_n, (Ek_{n-1})) = (\partial_0 g_n, k_{n-1}, (b_{n-1}))
\]

\[
\partial_0(g_n, (s_i^1 Ek_{n-i-1})) = (\partial_0 g_n, (s_i^1 (Ek_{n-i-1})) (i \geq 1).
\]

It will first be shown that the homology groups of \( T \) are trivial. This will be done by giving a contracting homotopy \( S \) for the chain complex \( C(T) \).

Lemma 3. Let \( G \) be the free group on generators \( x_\alpha \). Then the integral group ring \( ZG \) has as basis (over \( Z \)) the elements \( gx_\alpha - g \), where \( g \) ranges over all elements of \( G \); together with the element 1.

The proof is not difficult. Now define \( S \) by the rules

\[
S(l_n, (b_n)) = \begin{cases} 
0 & \text{(n even)} \\
(l_{n+1}, (b_{n+1}) & \text{(n odd)}
\end{cases}
\]

\[
S[(g_n, k_n, (b_n)) - (g_n', (b_n'))]
\]

124
\[ (10) \]
\[ S[ (g_n, (s_0^{-1}E_k n^{-1}r)) - (g_n, (b_n^{-1}r)) ] \]
\[ = \sum_{i=0}^{n} (-1)^i [(s_i g_n, (s_0^i E_k n^{-1}r)) - (s_i g_n, (b_n^{-1}r)) ] \]
\[ = \sum_{j=r}^{n} (-1)^j [(s_j g_n, (s_0^j E_k n^{-1}r)) - (s_j g_n, (b_n^{-1}r)) ] \]

where \( g_n \) ranges over all elements of the group \( FK_n \).

It follows easily from Lemma 3 that the elements for which \( S \) has been defined form a basis for \( C(T) \), providing that \( k_n \), \( k_n^{-1}r \) are restricted to elements other than \( b_n \), \( b_n^{-1}r \). However, the above rules reduce to the identity \( 0 = 0 \) if we substitute \( k_n = b_n \) or \( k_n^{-1}r = b_n^{-1}r \). This shows that \( S \) is well defined.

The necessary identity \( Sd + dS = 1 - \varepsilon \), where
\[ dx_n = \sum_{i=0}^{n} (-1)^i \partial_i x_n \]
and where \( \varepsilon: C(T) \to C(T) \) is the augmentation
\[ (\varepsilon \sum \alpha_i (g_0, b_0) = \sum \alpha_i (1, b_0)) \]
can now be verified by direct computation. Since this computation is rather long it will not be given here.

Proof that \( \left| T \right| \) is simply connected. A maximal tree in the CW-complex \( \left| T \right| \) will be chosen. Then \( \pi_1 (\left| T \right|) \) can be considered as the group with one generator corresponding to each 1-simplex not in the tree, and one relation corresponding to each 2-simplex.

As maximal tree take all 1-simplices of the form \((s_0 g_0, (E_k 0))\). Then as generators of \( \pi_1 (\left| T \right|) \) we have all elements \((g_1, (E_k 0))\) such that \( g_1 \) is non-degenerate. The relation \( \partial_1 x = (\partial_2 x) \cdot (\partial_0 x) \) where \( x = (s_1 g_1, (s_0 E_k 0)) \) asserts that
\[ (g_1, (E_k 0)) = (g_1, (b_1)) \cdot (s_0 \partial_0 g_1, (E_k 0)) \]
From the 2-simplex \((s_0 g_1, (E k_1))\) we obtain

\[
(g_1, (E \partial_0 k_1)) = (s_0 \partial_1 g_1, (E \partial_1 k_1)). (g_1 k_1, (b_1)) = (g_1 k_1, (b_1)).
\]

Combining these two relations we have

\[
(g_1, (b_1)) = (g_1 k_1, (b_1)),
\]

from which it follows easily that

\[
(g_1, (b_1)) = 1
\]

for all \(g_1\). In view of the first relation, this shows that \(|T|\) is simply connected, and completes the proof of theorem 1.

The following theorem shows that FK is essentially unique.

**Theorem 2.** Any principal bundle over \(E K\) with any group complex \(G\) as fibre is induced from the above bundle by a homomorphism \(FK \to G\).

**Proof.** We may assume that this bundle is a twisted cartesian product with twisting function \(\tau: (E K)_{n+1} \to G_n\). Define the homomorphism \(\overline{\tau}: FK \to G\) by \(\overline{\tau}(k_n) = \tau(E k_n)\). Since \(\overline{\tau}(b_n) = \tau(E b_n) = \tau(s_0(b_n)) = 1_n\) this defines a homomorphism. It is easy to verify that \(\overline{\tau}\) commutes with the face and degeneracy operations, and induces a map between the two twisted cartesian products.
Corollary. If $G$ is also a loop space for $EK$ then there is a homomorphism $FK \rightarrow G$ inducing an isomorphism between the Pontrjagin rings.

This follows easily using [7], IV Theorem B.

Analogues of theorems 1 and 2 for the construction $F^+(K)$ can be proved using exactly the same formulas. The following shows the relationship between $F^+(K)$ and the construction of James.

**Lemma 4.** If $K$ is countable then the realization $|F^+K|$ is homeomorphic to the reduced product of $|K|$.

In fact the product $(k_n, k'_n, k''_n, \ldots) \rightarrow k_n \cdot k'_n \cdot k''_n \cdot \ldots$ maps $K \times \ldots \times K$ into $F^+K$. Taking realizations we obtain a map $|K| \times \ldots \times |K| \rightarrow |F^+K|$. From these maps it is easy to define a map of the reduced product of $|K|$ into $|F^+K|$, and to show that it is a homeomorphism.

3. A theorem of Hilton

If $A$, $B$ are semi-simplicial complexes with base points $a_0$, $b_0$ let $A \vee B$ denote the subcomplex $A \times \{b_0\} \cup \{a_0\} \times B$ of $A \times B$. Let $A \times B$ denote the complex obtained from $A \times B$ by collapsing $A \vee B$ to a point. The notation $A^{(k)}$ will be used for the $k$-fold 'collapsed product' $A \times \ldots \times A$.

The free product $G \ast H$ of two group complexes is defined by $(G \ast H)_n = G_n \ast H_n$. There is clearly a canonical isomorphism between the group complexes $F(A \vee B)$ and $FA \ast FB$. 

127
Lemma 5. The complex $F(A \vee B)$ is isomorphic (ignoring group structure) to $FA \times F(B \vee (B \times FA))$.

In fact we will show that $F(A \vee B)$ is a split extension:

$$I - F(B \vee (B \times FA)) - F(A \vee B) - FA - I.$$

The collapsing map $A \vee B \sim A$ induces a homomorphism $c'$ of $F(A \vee B)$ onto $FA$. Denote the kernel of $c'$ by $F'$. The inclusion $A \hookrightarrow A \vee B$ induces a homomorphism $i'\colon FA \to F(A \vee B)$. Since $c'i'$ is the identity it follows that $F(A \vee B)$ is a split extension of $F'$ by $FA$.

We will determine this kernel $F'_n$ for some fixed dimension $n$. Let $a, b, \phi$ range over the $n$-simplexes other than the base point of $A, B,$ and $FA$ respectively. Then $F(A \vee B)_n$ is the free group $\{a, b\}$ and $F'_n$ is the normal subgroup generated by the $b$. By the Reidemeister-Schreier theorem (see [8]) $F'_n$ is freely generated by the elements $w(a)bw(a)^{-1}$ where $w(a)$ ranges over all elements of the free group $\{a\} = FA_n$. Thus

$$F'_n = \{b, \phi b \phi^{-1}\}.$$ 

Now setting $[b, \phi] = b \phi b^{-1} \phi^{-1}$ and making a simple Tietze transformation (see for example [1]) we obtain

$$F'_n = \{b, [b, \phi]\}.$$ 

Identify $[b, \phi]$ with the simplex $b \times \phi$ of $B \times F(A)$. Then we can identify $F'_n$ with $F(B \vee (B \times FA))$. Since this identification commutes with face and degeneracy operations, this proves Lemma 5.

128
Lemma 6. The group complex $F(B \ltimes FA)$ is isomorphic to

$$F((B \ltimes A) \vee (B \ltimes A \ltimes FA)).$$

The inclusion $A \to FA$ induces a homomorphism

$$F(B \ltimes A) \to F(B \ltimes FA).$$

A homomorphism

$$F(B \ltimes A \ltimes FA) \to F(B \ltimes FA)$$

is defined by

$$b \ltimes a \ltimes \phi = (b \ltimes a)(b \ltimes \phi a)^{-1}(b \ltimes \phi).$$

(This is motivated by the group identity $[[b, a], \phi] = [b, a]$

$[b, \phi a]^{-1}[b, \phi].$)

Combining these we obtain a homomorphism

$$F(B \ltimes A) \ltimes \tau F(B \ltimes A \ltimes FA) \to F(B \ltimes FA)$$

which is asserted to be an isomorphism.

Using the same notation as in Lemma 5 and identifying $b \ltimes a \ltimes \phi$ with $[[b, a], \phi]$ it is evidently sufficient to prove the following.

Lemma 7. In the free group $\{a, b\}$ the subgroup freely generated by the elements $[b, \phi]$ is also freely generated by the elements $[b, a]$ and $[[b, a], \phi]$. 

129
The proof consists of a series of Tietze transformations. Details will not be given.

As a consequence of Lemma 6 we have:

**Lemma 8.** For each $m$ the group complex $F(B \times FA)$ is isomorphic to

$$F(B \times A) \ast F(B \times A \ast A) \ast \ldots \ast F(B \times A^{(m)}) \ast F(B \times A^{(m)} \times FA).$$

Proof by induction on $m$. For $m = 1$ this is just Lemma 6. Given this assertion for the integer $m - 1$ it is only necessary to show that $F(B \times A^{(m-1)} \times FA)$ is isomorphic to $F(B \times A^{(m)} \ast F(B \times A^{(m)} \ast FA)$, but this follows immediately from Lemma 6 by substituting $B \times A^{(m-1)}$ in place of $B$.

**Theorem 3.** If $A$ and $B$ are semi-simplicial complexes with $A$ connected, then there is an inclusion homomorphism

$$F(\bigvee_{i=1}^{\infty} B \times A^{(i)}) \rightarrow F(B \times F(A))$$

which is a homotopy equivalence.

Proof. Every element of $F(\bigvee_{i=1}^{\infty} B \times A^{(i)})$ is already contained in

$$F(\bigvee_{i=1}^{\infty} B \times A^{(i)}) = F(B \times A) \ast \ldots \ast F(B \times A^{(m)})$$
where \( n = n_1 + \ldots + n_r \), \( \delta = \text{GCD}(n_1, \ldots, n_r) \).

**Proof.** For \( n = 1, 2, 3, \ldots \), define complexes \( A_i \) to be called 'basic products of weight \( n \)' as follows, by induction on \( n \).

The given complexes \( A_1, \ldots, A_r \) are the basic products of weight 1. Suppose that

\[
A_1, \ldots, A_r, \ldots, A_\alpha
\]

are the basic products of weight less than \( n \). To each \( i = 1, \ldots, r, \ldots, \alpha \) assume we have defined a number \( e(i) < i \), where \( e(1) = \ldots = e(r) = 0 \). Then as basic products of weight \( n \) take all expressions \( A_i \ast A_j \) where weight \( A_i \) + weight \( A_j = n \) and \( e(i) \leq j < i \). Call these new complexes \( A_\alpha+1, \ldots, A_\beta \) in any order. If \( A_h = A_i \ast A_j \), define \( e(h) = j \). (For this discussion we must consider complexes such as \( (A \ast B) \ast C \) and \( A \ast (B \ast C) \) to be distinct!) This completes the construction of the \( A_i \).

For each \( m \geq 1 \) define

\[
R_m = F(\bigvee_{h \geq m} A_h) \quad e(h) < m
\]

Thus \( R_1 = F(A_1 \vee \ldots \vee A_r) \).

**Lemma 9.** There is a homotopy equivalence

\[
F(A_m) \times R_{m+1} \subset R_m.
\]
for some \( m \). Hence by Lemma 8 it may be identified with an element of \( F(B \times FA) \). Since \( A \) is connected, the 'remainder term' \( B \times A^{(m)} \times FA \) has trivial homology groups in dimensions less than \( m \). From this it follows easily that the above inclusion induces isomorphisms of the homotopy groups in all dimensions.

**Remark.** The complex \( B \) may be eliminated from Theorem 3 by taking \( B \) as the sphere \( S^0 \), and noting the identity \( S^0 \times K = K \).

Combining theorem 3 with Lemma 5 we obtain the following

**Corollary.** If \( A \) is connected then there is a homotopy equivalence

\[
F(A) \times F(\bigvee_{i=0}^\infty B \times A^{(i)}) \subset F(A \vee B).
\]

This corollary will be the basis for the following.

**Theorem 4.** Let \( A_1, \ldots, A_r \) be connected complexes. Then \( F(A_1 \vee \ldots \vee A_r) \) has the same homotopy type as a weak infinite cartesian product \( \prod_{i=1}^\infty F(A_i) \) where each \( A_i, i > r, \) has the form

\[
A_1^{(n_1)} \times \ldots \times A_r^{(n_r)}.
\]

The number of factors of a given form is equal to the Witt number

\[
\phi(n_1, \ldots, n_r) = \frac{1}{n} \sum_{d|\delta} \frac{\mu(d)(n/d)!}{(n_1/d)! \ldots (n_r/d)!}
\]

131
Note that $R_m = F(A_m \vee B)$, where $B = \bigvee_{h > m} A_h$.

By the corollary to theorem 3 there is a homotopy equivalence

$$(F(A_m) \times F(\bigvee_{i=0}^{\infty} B \times A_m^{(i)})) \subset F(A_m \vee B) = R_m.$$ 

Substituting in the definition of $B$ and using the distributive law

$$(A \vee B) \times C = (A \times C) \vee (B \times C),$$

the second factor of the first expression becomes

$$F(\bigvee_{i=0}^{\infty} \bigvee_{h > m} A_h \times A_m^{(i)}).$$

But (filling in parentheses correctly) this is just

$$F(\bigvee_{h > m} A_h) = R_{m+1},$$

which proves Lemma 9.

Now it follows by induction that there is a homotopy equivalence

$$F(A_1) \times F(A_2) \times \ldots \times F(A_m) \times R_{m+1} \subset R_1 =$$

$$F(A_1 \vee \ldots \vee A_r).$$

This defines an inclusion of the weak infinite cartesian product
\( \Pi_1^\infty F(A_i) \) into \( R_1 \). Since \( A_1, \ldots, A_r \) are connected, it follows easily that the 'remainder terms' \( R_m \) are \( k \)-connected where \( k \to \infty \) as \( m \to \infty \). From this it follows that the above inclusion map induces isomorphisms of the homotopy groups in all dimensions. This proves the first part of theorem 4.

Let \( \phi(n_1, \ldots, n_r) \) denote the number of \( A_h \) having the form \( A_1^{(n_1)} \times \cdots \times A_r^{(n_r)} \). To compute these numbers consider the free Lie ring \( L \) on generators \( \alpha_1, \ldots, \alpha_r \). Corresponding to each 'basic product' \( A_h = A_i \times A_j \) define an element \( \alpha_h = [\alpha_i, \alpha_j] \) of \( L \), for \( h = r+1, r+2, \ldots \). Then the elements \( \alpha_h \) obtained in this way are exactly the standard monomials of M. Hall [2] and P. Hall [3]. M. Hall has proved that these elements form an additive basis for \( L \).

The number of linearly independent elements of \( L \) which involve each of the generators \( \alpha_1, \ldots, \alpha_r \) a given number \( n_1, \ldots, n_r \) of times has been computed by Witt [9]. Since his formula is the same as that in theorem 4, this completes the proof.

In conclusion we mention one more interesting consequence of theorem 3.

**Theorem 5.** If \( A \) is connected then the complex \( EFA \) has the same homotopy type as \( \bigvee_{i=1}^\infty \text{EA}^{(i)} \).

The proof is based on the following lemma, which depends on Theorem 1.

**Lemma 10.** If \( A \) is connected, there is a homotopy equivalence \( \text{EA} \subset \text{WFA} \).
In fact the inclusion is defined by \((s_0^i Ea_n) = s_0^i(a_{n-1}, l_{n-1}, \ldots, l_0)\). It is easily verified that this is a map, and that it induces a map of the twisted cartesian product \(T\) into the twisted cartesian product \(W\). Since both total spaces are acyclic, it follows from [7], IV Theorem A that the homology groups of \(EA\) map isomorphically into those of \(WF\). Since both spaces are simply connected, this completes the proof of Lemma 10.

Now from Theorem 3 we have a homotopy equivalence

\[
WF(\sim_{i=1}^{\infty} A^{(i)}) \subset WF(A).
\]

In view of Lemma 10, and the identity

\[
E(A \vee B) = EA \vee EB,
\]

this completes the proof.

References


The new theorem proposes a novel approach to killing homotopy.