11. (a) Embed $\mathbb{R}^n$ into $\mathbb{R}^{n+1}$ as the vectors with last coordinate equal to 0. This defines embeddings $S^{n-1} \hookrightarrow S^n$ and also $\mathbb{R}P^{n-1} \hookrightarrow \mathbb{R}P^n$. Write $S^\infty$ and $\mathbb{R}P^\infty$ for the spaces obtained as the direct limit of these sequences.

(i) Define a CW structure on each sphere making $S^{n-1}$ the $(n-1)$-skeleton of $S^\infty$, and on each projective space making $\mathbb{R}P^{n-1}$ the $(n-1)$-skeleton of $\mathbb{R}P^\infty$.

(ii) Show that the double covers $S^{n-1} \rightarrow \mathbb{R}P^{n-1}$ combine to give a double cover $S^\infty \rightarrow \mathbb{R}P^\infty$.

(iii) Determine the homotopy groups of these two spaces.

(b) Carry out the analogous story when $\mathbb{R}$ is replaced by $\mathbb{C}$.

12. Let $\omega \in \pi_1(S^1 \vee S^2)$ and $\alpha \in \pi_2(S^1 \vee S^2)$ be represented by the inclusion of the two spheres into the wedge. Form a new CW complex $X$ by attaching a 3-cell by means of a map representing the homotopy class $2\alpha - \omega \cdot \alpha \in \pi_2(S^1 \vee S^2)$. Show that the inclusion of $S^1$ into $X$ induces isomorphisms in $\pi_1$ and in homology, but that $X$ is not weakly equivalent to the circle.

[So no simple adjustment to the Whitehead theorem will work. Notice however that the map on universal covers is not an isomorphism in homology.]

13. Identify each of the spaces $\tau_{>2}\mathbb{C}P^n$ and $\tau_{\leq2}S^2$ with known CW complexes (up to homotopy type, of course).

14. (a) Let $N < G$ be a normal subgroup, with quotient group $H$. Show that there is a fibration $K(G,1) \rightarrow K(H,1)$ with fiber weakly equivalent to $-K(N,1)$.

(b) Suppose that $G$ is abelian. Then the same argument gives us a fibration $K(G,n) \rightarrow K(H,n)$ with fiber $K(N,n)$. But show also that there is a fibration $K(N,n) \rightarrow K(H,n - 1)$, and a fibration $K(H,n) \rightarrow K(N,n + 1)$ with fiber $K(G,n)$.

For example, what is the homotopy fiber of the map $\mathbb{C}P^\infty \rightarrow \mathbb{C}P^\infty$ represented by twice a generator of $H^2(\mathbb{C}P^\infty)$?

15. Let $Y$ be a simple space and $N$ an integer, and suppose that $N\pi_*(Y) = 0$. Let $(X,A)$ be a relative CW complex and assume that $H_*(X,A;\mathbb{F}_p) = 0$ whenever the prime $p$ divides $N$. Show that the restriction map $[X,Y] \rightarrow [A,Y]$ is bijective.