1) (a) \[
\int_{-\infty}^{x} \delta(y - 2) \, dy
\]
If \(x < 2\), the region of integration does not include singularity, \(\int_{-\infty}^{x} \delta(y - 2) \, dy = 0\).

If \(x > 2\), the region of integration includes the singularity, \(\int_{-\infty}^{x} \delta(y - 2) \, dy = 1\).

So, \(\int_{-\infty}^{x} \delta(y - 2) \, dy = H(x - 2)\).

(b) \[
\int_{-\infty}^{x} \delta(y - 3) \, dy = H(x - 3) \quad \text{and} \quad \int_{-\infty}^{x} \delta(y + 2) \, dy = H(x + 2)
\]
So, \[
\int_{-\infty}^{x} \left(\delta(y - 3) - \frac{1}{2}\delta(y + 2)\right) \, dy = H(x - 3) - \frac{1}{2}H(x + 2).
\]

2) (a) Constant force down: \(f = -1\)
(b) Slope is 0 on the right, constant except at x=1/2, 3/4. Force is up at x=1/2, 3/4. 

(c) Force is up for x<1/2, force is down for x>1/2

To determine that the slope at x=0 equals zero: 
Integrate both sides from 0 to 1:
\[
\int_0^1 - \frac{d^2u}{dx^2} = \int_0^1 f
\]
\[
\frac{du}{dx} (0) - \frac{du}{dx} (1) = 0
\]
Since \(\frac{du}{dx} (1) = 0\), \(\frac{du}{dx} (1) = 0\) as well.

3) \[
- \frac{d^2u}{dx^2} = \sin \frac{\pi x}{L}
\]
\[
u(0) = 0 \\
u(L) = 0
\]
Integrating:
\[
u(x) = \frac{L^2}{\pi^2} \sin \frac{\pi x}{L} + cx + d
\]
\[
u(0) = 0 \text{ means } d = 0
\]
\[
u(L) = 0 \rightarrow \nu(L) = \frac{L^2}{\pi^2} \sin \pi + cL = cL = 0 \text{ means } c = 0.
\]
Solution:
\[
u(x) = \frac{L^2}{\pi^2} \sin \frac{\pi x}{L}
\]
4)

\[- \frac{d^2 u}{dx^2} = \delta(x - L/2)\]

\[u(0) = 0\]

\[u'(L) = 0\]

Integrating:

\[u(x) = ax + b \quad \text{for } x < \frac{L}{2}\]

\[u(x) = cx + d \quad \text{for } x > \frac{L}{2}\]

The two sides of the function are linear because \(\frac{d^2 u}{dx^2} = 0\) away from \(x = L/2\).

\(u(0) = 0\) means \(b = 0\), \(u'(L) = 0\) means \(c = 0\).

Now:

\[u(x) = ax \quad \text{for } x < \frac{L}{2}\]

\[u(x) = d \quad \text{for } x > \frac{L}{2}\]

Apply continuity at \(x = \frac{L}{2}\):

\[a \frac{L}{2} = d\]

Apply the jump condition:

\[-\left.\frac{du}{dx}\right|_{L/2} = 1\]

\[-(c - a) = 1\]

Since \(c = 0, a = 1\). Since \(aL/2 = d, d = L/2\)

Solution:

\[u(x) = x \quad \text{for } x < \frac{L}{2}\]

\[u(x) = \frac{L}{2} \quad \text{for } x > \frac{L}{2}\]
5) (a) i)

\[
\frac{y_{n+1} - y_n}{\Delta t} = Ay_n
\]
\[
y_{n+1} = y_n + A\Delta t y_n
\]

\[
y_{n+1} = (I + A\Delta t)y_n
\]

ii)

\[
\frac{y_{n+1} - y_n}{\Delta t} = Ay_{n+1}
\]
\[
y_{n+1} - A\Delta t y_{n+1} = y_n
\]
\[
(I - A\Delta t)y_{n+1} = y_n
\]

\[
y_{n+1} = (I - A\Delta t)y_n
\]

iii)

\[
\frac{y_{n+1} - y_n}{\Delta t} = \frac{Ay_{n+1} + Ay_n}{2}
\]
\[
y_{n+1} - \frac{\Delta t}{2}Ay_{n+1} = y_n + \frac{\Delta t}{2}Ay_n
\]

\[
y_{n+1} = \left(I - \frac{A\Delta t}{2}\right)\left(I + \frac{A\Delta t}{2}\right)y_n
\]

b-d)

\[
A=\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix};
\]
\[
y_0=\begin{bmatrix} 1 \\ 0 \end{bmatrix}; \ \text{initial } y\text{-value, } y(0)
\]
\[
dt=0.01; \ \text{step size}
I=\text{eye}(2);
\]
\[
[t, y]=\text{ode45}(@(t,y)A*y,[0;1],[1;0]); \ \text{arguments: function, timespan, } y(0)
\]
\[
y_1_{\text{forward}}=(I+A\cdot\text{dt})^{(1/\text{dt})}y_0; \ \text{where } 1/\text{dt} \text{ is the number of steps to } y(1)
\]
\[
y_1_{\text{trap}}=((I-A\cdot\text{dt}/2)\backslash(I+A\cdot\text{dt}/2))^{(1/\text{dt})}y_0;
\]
\[
y_1_{\text{ode45}}=y(\text{end},:);
\]

Results:

\[
y_{1\_\text{forward}} =
\begin{bmatrix} 0.3660 \\ 0.3697 \end{bmatrix}
\]
\[
y_{1\_\text{trap}} =
\begin{bmatrix} 0.3679 \\ 0.3679 \end{bmatrix}
\]
\[
y_{1\_\text{ode45}} =
\begin{bmatrix} 0.3679 \\ 0.3679 \end{bmatrix}
\]