Problem set 6, due october 31

The following exercises are taken from the book, references are from Ross, Edition 9, chapter 5. Due Friday October 31 at the beginning of lecture.

Problem 1 (Problem 24) The lifetimes of interactive computer chips produced by a certain semiconductor manufacturer are normally distributed with parameters $\mu = 1.4 \times 10^6$ hours and $\sigma = 3 \times 10^5$ What is the approximate probability that a batch of 100 chips will contain at least 20 whose lifetimes are less than 1.8×10^6 ?

Problem 2 (Problem 26) Two types of coins are produced at a factory : a fair coin and a biased one that comes up head 55 percent of the time. We have one of these coins but do not know whether it is fair or biased. In order to ascertain which type of coin we have we shall perform the following statistical test: We shall toss the coin 1000 times. If the coin lands on heads 525 or more times, we shall conclude the coin is biased, and otherwise that it is fair. If the coin is actually fair, what is the probability that we reach a false conclusion ? What would it be if the coin were biased ?

Problem 3(Problem 32) : The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = 1/2$. What is

(1) the probability that a repair time exceeds 2 hours?
(2) the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 9 hours?

Problem 4 (Theoretical Exercise 11) Let Z be a standard normal random variable and let g be a differentiable function with derivative g'.

(1) Show that $\mathbb{E}[g'(Z)] = \mathbb{E}[Zg(Z)]$.
(2) Show that $\mathbb{E}[Z^{n+1}] = n\mathbb{E}[Z^{n-1}]$
(3) Find $\mathbb{E}[Z^6]$.

Problem 5 (Theoretical Exercise 13) The median of a continuous random variable having distribution function F is that value m such that $F(m) = 1/2$. Find the median of X if X is

(1) uniformly distributed over (a, b);
(2) normal with parameters μ, σ,
(3) exponential with rate λ.

Problem 6(Theoretical Exercise 29) Let X be a continuous random variable having cumulative distribution function F. Define the random variable Y by $Y = F(X)$. Show that Y is uniformly distributed over $(0, 1)$.

Problem 7 At time zero, a single bacterium in a dish divides into two bacteria. This species of bacteria has the following property: after a bacterium B divides into two new bacteria B_1 and B_2, the subsequent length of time until each B_i divides is an exponential random variable of rate $\lambda = 1$, independently of everything else happening in the dish.

(1) Compute the expectation of the time T_n at which the number of bacteria reaches n
(2) Compute the variance of T_n
(3) Are both of the answers above unbounded, as functions of n ? Give a rough numerical estimate of the values when $n = 10^{40}$.