Problem set 5, due october 24

The following exercises are taken from the book, references are from Ross, Edition 9, chapter 4 and 5.

Due Friday October 24 at the beginning of lecture.

Chapter 4:

Problem 1(Problem 71) Consider a roulette wheel consisting of 38 numbers 1 through 36, and double 0. If Smith always bets that the outcome will be one of the numbers 1 through 12, what is the probability that

(1) Smith will lose his first 5 bets;
(2) The mean number of time he will have won if he plays 10 times;
(3) his first win will occur on his fourth bet;

Problem 2(Problem 70) At time 0 a coin that comes up heads with probability p is flipped and falls to the ground. Suppose it lands on heads. At times chosen according to a Poisson process $(N_t, t \geq 0)$ with rate λ, the coin is picked up and flipped. (This means that at the random times $T_k = \inf\{t : N_t = k\}$ the coin is flipped. Between these times the coin remains on the ground.) What is the probability that the coin is on its head side at time t ? *Hint:* What would be the conditional probability if there were no additional flips by time t (that is $T_1 > t$) , and what would it be if there were additional flips by time t (that is $T_1 < t$) ?

Problem 3 (Theoretical exercise 19) Show that if X is a Poisson random variable with parameter λ

$$\mathbb{E}[X^n] = \lambda \mathbb{E}[(X+1)^{n-1}]$$

Use this result to compute $\mathbb{E}[X^3]$.

Problem 4(Theoretical exercise 16 and 18) Let X be a Poisson random variable with parameter λ.

(1) Show that $P(X = i)$ increases and then decreases. Give the value of i so that it is maximum.
(2) What value of λ maximizes $P(X = k), k \geq 0$?

Problem 5 (Theoretical Exercise 25) Suppose that the number of events that occur in a specified

time is a Poisson random variable with parameter λ. If each event is "counted" with probability p, independently of every other event, show that the number of events that are counted is a Poisson random variable with parameter $\lambda \times p$. Also, give an intuitive argument as to why this should be so. As an application of the preceding result, suppose that the number of distinct uranium deposits in a given area is a Poisson random variable with parameter $\lambda = 10$. If, in a fixed period of time, each deposit is discovered independently with probability 1/50, find the probability that (a) exactly 1, (b) at least 1, and (c) at most 1 deposit is discovered during that time.

Chapter 5:

Problem 6 (Problem 4) The probability density function of X, the lifetime of a certain type of electronic device (measured in hours) is given by

$$f(x) = \begin{cases} \frac{10}{x^2} & x > 10 \\ 0 & 0 \leq x \leq 10 \end{cases}$$

(1) Find $P(X > 20)$
(2) What is the cumulative distribution function of X?
(3) What is the probability that of 6 such types of devices, at least 3 will function at least 15 hours ? What assumptions are you making ?

Problem 7 (Problem 11) A point is chosen at random on a line segment of length L. Interpret this statement, and find the probability that the ratio of the shorter to the longer segment is less than $1/4$.