Problem set 3, due september 26

The following exercises are taken from the book, references are from Ross, Edition 9, chapter 3. Due Friday september 26 at the beginning of lecture.

• **Problem 1(Problem 29)** There are 15 tennis balls in a box, of which 9 have not previously been used. Three of the balls are randomly chosen, played with, and then returned to the box. Later another 3 balls are randomly chosen from the box. Find the probability that none of these balls has ever been used.

• **Problem 2 (Problem 33)** On rainy days, Joe is late to work with probability 0.3, on non rainy days, he is late with probability $0, 1$. With probability 0.7 it will rain tomorrow.
  (1) Find the probability that Joe is early tomorrow.
  (2) Given that Joe was early, what is the conditional probability that it rained ?

• **Problem 3 (Problem 61)** Genes relating to albinism are denoted by A and a. Only this people who receive gene a from both parents will be albino. Persons having the gene pair (A, a) are normal in appearance and, because they can pass on the trait to their offsprings, are called carriers. Suppose that a carrier can give the gene A or a with equal probability. Suppose that a normal couple has two children, exactly one of whom is an albino.
  (1) What is the probability that the non albino child has the gene a?
  (2) Suppose that the nonalbino child mates with a person who is a carrier. What is the probability that their first offspring is an albino? What is the conditional probability that their second offspring is an albino given that their first born is not ?

• **Problem 4 (Problem 84)** An urn contains 12 balls, of which 4 are white. Three players A, B, C successively draw from the urn: A first, then B then C then A etc. The winner is the first one to draw a white ball. Find the probability of winning for each player if
  (1) each ball is replaced after drawn,
  (2) the balls that are withdrawn are not replaced.

•**Exercise 5** Suppose that a fair coin is tossed infinitely many times, independently. Let X_i denote the outcome of the ith coin toss (an element of $\{H, T\}$) Compute the probability that:
  (1) $X_i = H$ for all positive integers i
  (2) The pattern HHTTHHTT occurs at some point in the sequence.

• **Exercise 6** An urn contains b black balls and r red balls. One of the balls is drawn at random, but when it is put back in the urn c additional balls of the same color are put in with it. Now suppose that we draw another ball. Show that the probability that the first ball drawn was black given that the second ball drawn was red is $b/(b + r + c)$.

• **Exercise 7 (Theoretical exercise 24)** A round-robin tournament of n contestants is a tournament in which each of the $\binom{n}{2}$ pairs of contestants play each other exactly once, with the outcome of any play being that one of the contestants wins and the other loses. For a
fixed integer $k, k < n$ a question of interest is whether it is possible that the tournament outcome is such that, for every set of k players, there is a player who beat each member of that set. Show that if

$$\binom{n}{k} [1 - (\frac{1}{2})^k]^{n-k} < 1$$

then such an outcome is possible. Hint: Suppose that the results of the games are independent and that each game is equally likely to be won by either contestant. Number the $\binom{n}{k}$ sets of contestants, and let B_i denote the event that no contestant beat all of the k players in the ith set. Then use Boole's inequality to bound $P(\cup_i B_i)$