Problem set 2, due september 22

The following exercises are taken from the book, references are from Ross, Edition 9, chapter two.

Due Monday september 22 at the beginning of lecture.

• **Problem 1(Problem 6)** A hospital administrator codes incoming patients suffering gunshot wounds according to whether they have insurance (coding 1 if they do and 0 if they do not) and according to their condition, which is rated as good (g), fair (f), or serious (s). Consider an experiment that consists of the coding of such a patient.

(1) Give the sample space of this experiment.
(2) Let A be the event that the patient is in serious condition. Specify the outcomes in A.
(3) Let B be the event that the patient is uninsured. Specify the outcomes in B.
(4) Give all the outcomes in the event $B^c \cup A$.

• **Problem 2 (Problem 42)** Two dice are thrown n times in succession. Compute the probability that double 6 appears at least once. How large need n to be to make this probability at least $1/2$?

•**Problem 3 (Problem 52)** A closet contains 10 pairs of shoes. If 8 shoes are selected, what is the probability that there will be

(1) No complete pair ?
(2) Exactly one complete pair ?

• **Problem 4 (Problem 48)** Given 20 people, what is the probability that, among the 12 months in the year, there are 4 months containing exactly 2 birthdays and 4 containing exactly 3 birthdays?

• **Exercise 5 (Theoretical Exercise 10)** Prove that

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E^c FG) - P(EF^c G) - P(EFG^c) - 2P(EFG) \,.$$

•**Exercise 6 (Exercises 11 and 16)**

(1) Show that if E, F are two subsets, $P(E \cap F) \geq P(E) + P(F) - 1$
(2) Deduce by induction that if $E_1, \ldots, E_n$ are n subsets , then

$$P(E_1 \cap E_2 \cap \cdots \cap E_n) \geq P(E_1) + P(E_2) + \cdots P(E_n) - n + 1$$

• **Exercise 7 (Theoretical Exercise 20)** Consider an experiment whose sample space consists of a countably infinite number of points. Show that not all points can be equally likely. Can all points have a positive probability of occurring?

• **Exercise 8** A deck of cards contains 52 cards with labels $1, 2 \ldots, 52$ and a player is given 10 of these cards.

• Find the probability that 4 cards have odd numbers and 6 have even numbers.
• Find the probability that each of the numbers on the ten cards ends with a different digit.
• Find the probability that all numbers are a multiple of 4.