

An Analysis of the Finite Element Method

Second Edition

Gilbert Strang and George Fix[†]

INTRODUCTION TO THE SECOND EDITION

FOREWORD TO THE 1997 EDITION

PREFACE

1	AN INTRODUCTION TO THE THEORY	1
1.1	The Basic Ideas	1
1.2	A Two-point Boundary-value Problem	3
1.3	The Variational Form of the Problem	8
1.4	Finite Difference Approximations	16
1.5	The Ritz Method and Linear Elements	24
1.6	The Error with Linear Elements	39
1.7	The Finite Element Method in One Dimension	51
1.8	Boundary Value Problems in Two Dimensions	63
1.9	Triangular and Rectangular Elements	74
1.10	Element Matrices in Two-dimensional Problems	90
2	A SUMMARY OF THE THEORY	101
2.1	Basis Functions for the Finite Element Spaces S^h	101
2.2	Rates of Convergence	105
2.3	Galerkin's Method, Collocation, and the Mixed Method	116
2.4	Systems of Equations; Shell Problems; Variations on the Finite Element Method	126
3	APPROXIMATION	136
3.1	Pointwise Approximation	136
3.2	Mean-square Approximation	143
3.3	Curved Elements and Isoparametric Transformations	156
3.4	Error Estimates	165

4	VARIATIONAL CRIMES	172
4.1	Violations of the Rayleigh-Ritz Code	172
4.2	Non-conforming Elements	174
4.3	Numerical Integration	181
4.4	Approximation of Domain and Boundary Conditions	192
5	STABILITY	205
5.1	Independence of the Basis	205
5.2	The Condition Number	209
6	EIGENVALUE PROBLEMS	216
6.1	Variational Formulation and the Min-max Principle	216
6.2	Some Elementary Examples	223
6.3	Eigenvalue and Eigenfunction Errors	228
6.4	Computational Techniques	236
7	INITIAL-VALUE PROBLEMS	241
7.1	The Galerkin-Crank-Nicolson Method for the Heat Equation	241
7.2	Stability and Convergence in Parabolic Problems	245
7.3	Hyperbolic Equations	251
8	SINGULARITIES	257
8.1	Corners and Interfaces	257
8.2	Singular Functions	263
8.3	Errors in the Presence of Singularities	266
8.4	Experimental Results	268
	BIBLIOGRAPHY	281
	INDEX OF NOTATIONS	297
	INDEX	303

PART 2 (2008)

9	FINITE ELEMENTS IN ONE DIMENSION	307
9.1	Weak Form with Linear Trial Functions	307
9.2	Cubic Splines and Fourth Order Equations	319
10	THE FINITE ELEMENT METHOD IN 2D AND 3D	329
11	ERRORS IN PROJECTIONS AND EIGENVALUES	349
12	MIXED FINITE ELEMENTS: VELOCITY AND PRESSURE	356
	APPENDIX A: DISCONTINUOUS GALERKIN METHODS	367
	APPENDIX B: FAST POISSON SOLVERS	391
	INDEX	401