Projects

Project Guidelines

(Essentially Taken from OCW)

You must write a mathematical paper approximately ten pages long. It must be written in LaTeX, and it should be written so that other people in the seminar can read and understand it. After the project has been submitted, you will give a short presentation based on your mathematical paper.

You must submit a one paragraph project proposal two months before the project is due. It should include at least one or two books or papers that you plan to use. You must then submit a draft about two weeks before the paper is due. I will make writing suggestions which can then be implemented in the final version of the paper.

- The paper should be somewhere around 10 pages in length. The length does not really matter, but it should be long enough to discuss something interesting.
- The paper should be LaTexed. You should let me know if you do not know LaTex.
- It should have a bibliography - even if there is only one reference.
- Write it so that other people in the seminar can read and understand it. In particular, define things, give motivation, etc.
- Don't put all of your equations in the middle of your text, rather, use "displayed equations" to make reading easy.
- Work out examples of what you write about and include these.
- Feel free to include figures.
- Be precise!

Grading

The proposal, draft, presentation, and paper will all receive a letter grade. For each day any of the above is late, the maximum letter grade you can receive for that portion decreases by one full letter grade. For example, if your proposal is one day late, your maximum proposal grade is a B. The total final project grade will be a weighted average of the four grades.

Project Ideas

Below is a list of some ideas for final project topics. Feel free to come up with your own ideas as well.

1. Algebraic topology and complex analysis. Homotopy invariance of path integrals and Cauchy's theorem. Computing degrees, homotopy theoretic proof of fundamental theorem of algebra.
2. Homology/cohomology. Although we're not covering it in our class, you could write about it in your final project. It comes in many (related) flavors:
    - Simplicial or polygonal homology - the most intuitive version - based on triangulations.
    - Singular homology - the most functorial - a little non-intuitive.
    - Cellular homology - formulated for CW complexes - we'll talk about these later.
    - de Rham cohomology - a differential geometric version using differential forms.
3. Jordon curve theorem. Let C be a closed simple (nonintersecting) curve in the plane. Theorem: the complement of the curve consists of two components, the "inside" and the "outside". Surprisingly tricky to prove. A related problem is "invariance of domain" - is R^n homeomorphic to R^m if n is not equal to m?
4. Handlebodies. We built surfaces using polygonal identification. There is another way to build surfaces and manifolds, by "attaching handles". This perspective is closely linked to "surgery theory".
5. Morse theory. A beautiful theory that decomposes manifolds using critical points of functions on them. Milnor wrote a very nice account of this. Related to "handlebodies".
6. Higher homotopy groups. The fundamental group is the study of maps of circles into your space. The higher homotopy groups study maps of n-spheres into your space. They are mysterious. Especially the higher homotopy groups of spheres.
7. Chain complexes, and chain homotopy. There is an algebraic version of homotopy where you replace spaces with chain complexes, and the notion of homotopy becomes chain homotopy.
8. Simplicial sets. Another combinatorial/algebraic way to model homotopy is the notion of a simplicial set. It is a triangulation, without the triangles!

9. Poincare homology sphere. Suppose that an n-manifold X has the same homology as the n-sphere. Is X a sphere? Poincare showed this was false by producing his "exotic homology sphere". He conjectured that if you additionally assumed that the fundamental group was trivial, then X was a sphere. This famous problem is known as the poincare conjecture, has only recently been solved for n = 3. The cases of large n were handled by Smale.
10. Galois theory and covering spaces. Galois theory feels like covering spaces, but the analogy goes farther than that. Two ideas:
    - Read Andreas Dress's new proof of the fundamental theorem of Galois theory. Very slick.
    - Learn about the algebraic version of covering spaces, so called etale maps. This requires some commutative algebra background.
11. Branched coverings. In complex analysis it is common to study things that are almost covering spaces, so called branched coverings. Riemann surfaces arise as branched coverings of projective space.
12. Hyperbolic space. Higher genus surfaces arise as quotients of group actions on negatively curved space, so-called hyperbolic space. Variants of this topic could involve learning about Teichmuller space, or Fuchsian groups.
13. Classifying spaces, principle fibrations. Galois covering spaces are examples of principle fibrations - these are bundles of discrete groups that lie over the space. By replacing the discrete groups with topological groups, you get the more general notion of a principle fiber bundle. A related topic is that of the classifying space. Another related topic is the notion of a Vector Bundle**.**
14. Universal covers for bad spaces. We will prove that universal covers exist for spaces which are "semi-locally simply connected". What if you have an arbitrary weird topological space. Daniel Biss wrote an amusing article that explains what you do.
15. Homology/cohomology of groups. There is a way to take the homology and cohomology of a group. This records important algebraic information about the group.
16. Seifert/Van Kampen using covering spaces. There apparently is a slick proof of the Seifert/Van Kampen theorem using covering spaces.
17. Categories, and colimits. Learn about the abstract notions of categories and functors. The universal property expressed in the Seifert/Van Kampen theorem is an instance of the notion of a "colimit" of a diagram.
18. Fundamental groupoid. This also ties into categories. The idea is rather than pick a basepoint, and define the fundamental group, use all points simultaneously. These pesky basepoint issues dissolve away, and the Seifert-Van Kampen theorem attains a cleaner statement.
19. Lens spaces. By our classification of surfaces, and our computation of their homotopy groups, we saw that two surfaces are homotopy equivalent if and only if they are homeomorphic. Is this true for arbitrary manifolds? The answer is no, and is proved using Lens spaces and Whitehead torsion.
20. Cobordism. A looser way to classify manifolds is up to cobordism: two manifolds are cobordant if they form the boundary of a manifold of one dimension higher.
21. Kuratowski's Theorem. Which graphs can be embedded in the plane? The complete answer is known.
22. Knot theory. There are a lot of possibilities here - fundamental groups of know complements, knot invariants, etc.
23. Arithmetic groups and trees. Bass-Serre theory gives a presentation for any group acting on a tree. These examples occur in the study of arithmetic groups. Serre wrote a beautiful book on the subject - the english translation is entitled "Trees".

Sample Student Projects

"Topology and economics" by Songzi Du (PDF) (Courtesy of Songzi Du. Used with permission.)

"Homology and cohomology" by Elleard Heffern (PDF) (Courtesy of Elleard Heffern. Used with permission.)