# Lecture Instructions

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<th>SES #</th>
<th>TOPICS</th>
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<tr>
<td>1</td>
<td>Organizational Meeting</td>
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<td>2</td>
<td>Susan Ruff</td>
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<td>3</td>
<td>n-manifolds and Orientability, Compact, Connected 2-manifolds</td>
<td><strong>Lecture 1 Gabriel (practice: Saturday 3 PM)</strong>&lt;br&gt;Define the notion of an n-manifold, and give examples. Discuss orientability&lt;br&gt;Some ideas: how exactly is $S^n$ seen to satisfy the definition of an n-manifold? You could give other examples - consulting other references perhaps - but Lecture 2 is devoted to talking about the examples which are compact surfaces, so try to not dwell on these. Non-examples could be discussed. Is the closed unit disk a manifold? Why or why not? What does orientability mean intuitively? Rigorously? How does the determinant condition fit into the Mobius band example? Is $S^n$ orientable for all n?</td>
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<td>4</td>
<td>Classification Theorem for Compact Surfaces, Triangulation</td>
<td><strong>Lecture 2 Nerse (practice: Saturday 2 PM)</strong>&lt;br&gt;Give examples of compact connected 2-manifolds. Important examples: 2-sphere (which will probably appear in Lecture 1), the torus, the projective plane, the Klein bottle&lt;br&gt;Some things to perhaps be addressed: What do these look like? (draw pictures) How do we give precise mathematical descriptions (gluing edges: the quotient topology, equations) - how can we use these precise mathematical descriptions to verify that these examples satisfy the definition of a manifold? Which ones are orientable, which ones are not? How can we visualize the projective plane? Building other examples: discuss the connect sum</td>
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<td><strong>Lecture 3 Manuel (practice: Monday 2 PM)</strong>&lt;br&gt;State the classification theorem of compact surfaces - our short-term goal is to prove it&lt;br&gt;Ideas: explain the canonical forms for our surfaces. Why do these give use the surfaces we think they do? How is the connect sum showing up in the polygons representation? How do we recognize orientability in these. Do many examples - what does a three holed torus connect sum a projective space look like in polygonal representation? Why are these things compact surfaces?</td>
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<td><strong>Lecture 4 Jacob (practice: Tuesday 2 PM)</strong>&lt;br&gt;Explain the notion of a triangulation. Then embark in the first step of the proof of the classification of surfaces: every compact surface can be viewed as the quotient of a disk with edges appropriately identified&lt;br&gt;Ideas: Give examples of triangulations, and non-examples. Try to conceptualize the elaborate argument in step 1, and give an example of it to get a feel for how it works</td>
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| 5     | Classification Theorem for Compact Surfaces (cont.), Euler Characteristic | **Lecture 5 Alex F.**  
Finish the proof of the Theorem 7.2 - every compact surface fits into the classification. (Then we just need to convince ourselves that these surfaces are all different)  
Ideas: These are some elaborate arguments - see if you can pick out the main points and summarize them for us. You may not have time to talk about lemma 7.1, but we can read about it if that's the case. An example would be great - probably from step 1 |
| 6     | Review of Group Theory, Homotopy and the Fundamental Group | **Lecture 7 Harrison**  
Review of group theory  
This material does not appear in Massey. This is supposed to be a review of the most basic notions of group theory. It would be great if all of the following concepts could be briefly described  
- Define what a group is  
- Examples - (Z,+), (R,+), (R\{0,*)  
- Multiplicative versus additive notation for Z  
- The cyclic groups of finite order  
- Subgroups  
- Group homomorphisms  
- Normal subgroups and quotients  
**Lecture 8 Alex A.**  
Define homotopy equivalence of paths. Why is it an equivalence relation? Define the product of paths. Show the product is associative. Define the inverse path, and show it is an inverse with respect to the path product. Define the fundamental group. Do not discuss the dependence of \( \pi_1 \) on the basepoint - that will be done in Lecture 9 |
| 7     | The Fundamental Group (cont.), Homotopy Equivalence and Homotopy Type | **Lecture 9 Nerses**  
Explain the dependence of the fundamental group on the basepoint. Describe the effect of a continuous mapping on the fundamental group. Show that it is a group homomorphism. Define the notion of homotopic maps, and relative homotopy. Explain theorem 4.1. Do not discuss the notion of a retract or deformation retract (that is for Lecture 12)  
Ideas: Explain these important notions intuitively, draw schematic pictures to accompany your abstract definitions. Discuss exercise 4.1. Give |
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<td>examples of homotopic maps, relatively homotopic maps, and homotopic maps which are not relatively homotopic</td>
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<td><em>Lecture 10 Gabriel</em></td>
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<td>Prove Lemma 8.1 and Theorem 8.2. Define the notions of homotopy equivalence and homotopy type. Deduce Theorem 8.3</td>
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<td>Ideas: give examples of homotopy equivalent spaces, and spaces which &quot;appear&quot; not homotopy equivalent. Is homotopy equivalence an equivalence relation. Why is it hard to show two spaces are not homotopy equivalent? Is the 1 point space equivalent to the (discrete) 2 point space? Look at exercise 8.2</td>
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<td><em>Lecture 11 Manuel</em></td>
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<td>8</td>
<td><em>Tuesday March 3</em></td>
<td>The Fundamental Group of a Circle, Retracts, Brower Fixed-Point Theorem</td>
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<td><em>Lecture 11 Manuel</em></td>
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<td>Prove the Very Important Theorem: pi_1(S^1) = Z</td>
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<td><em>Lecture 12 Jacob</em></td>
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<td>Define the notion of retract. Deduce that retracts give rise to monomorphisms and epimorphisms of fundamental groups. Define the notions of deformation retract, contractible, and simply connected. State and prove the Brower fixed point theorem</td>
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<td>Ideas: Give examples and non-examples of retracts, deformation retracts, simply connected spaces, contractible spaces</td>
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<td><em>Thursday March 5</em></td>
<td>Deadline for choosing a final project topic</td>
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<td>Weak Product of Groups, The Fundamental Group of a Torus, Free Abelian Groups</td>
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<td><em>Lecture 13 Harrison</em></td>
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<td>Define the product and weak product of groups. Discuss its &quot;universal property&quot; of the weak product (direct sum) of Abelian groups. At least state Theorem 7.1 of chapter 2, which says that the fundamental group of a product is the product of fundamental groups - and if there is time, prove it. Explain how this allows us to compute the fundamental group of the torus</td>
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<td><em>Lecture 14 Alex A.</em></td>
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<td>Explain the notion of a free abelian group, and how arbitrary abelian groups are quotients of free abelian groups by &quot;relations&quot;. Explain what Theorem 3.6 is saying. Give examples</td>
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<td>10</td>
<td><em>Tuesday March 10</em></td>
<td>Free Products, Free Groups, Presentations of Groups</td>
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<td><em>Lecture 15 Daniel</em></td>
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<td>Explain the free product, and what a free group is. Do not cover the commutator subgroup discussion at the end of section 5 - That will be covered in Lecture 16</td>
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<td><em>Lecture 16 Alex F.</em></td>
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<td>Discuss the notion of a presentation of a group - give examples. You can talk about the commutator subgroup (end of section 5) in connection with this</td>
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<tr>
<td>11</td>
<td>Thursday March 12</td>
<td>Discussion</td>
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| 12    | Siefert-Van Kampen Theorem and its Generalization | **Lecture 17 Gabriel**
State the Siefert-Van Kampen Theorem 2.1, and its generalization Theorem 2.2. Prove Lemma 2.3. This theorem has a rather abstract statement - what, intuitively, is it saying? (Consult Hatcher for a more palatable formulation, Theorem 1.20 of Chapter 1)

**Lecture 18 Grace**
Summarize the proof of Theorem 2.2. There are a lot of details here - try to pick out the main points |
| 13    | Applications of the Siefert-Van Kampen Theorem, Structure of the Fundamental Group of a Compact Surface | **Lecture 19 Manuel**
Prove Theorem 3.1. Compute the fundamental group of the "rose with n petals". Deduce the fundamental group of the n-punctured plane. Give an intuitive discussion of Lemma 3.2

**Lecture 20 Harrison**
Prove Theorem 4.1. Use it to give an alternative computation of the fundamental group of the torus (5.1). Compute the fundamental group of the projective plane (5.2) |
| 14    | Fundamental Groups on Closed Surfaces, Application to Knot Theory | **Nothing later builds upon the two talks for this day.**

**Lecture 21 Nerses**
Use the computations of the fundamental group of the torus and projective plane to compute the fundamental groups of the rest of the closed surfaces. This is Prop 5.1. If you have time, dwell upon what the group for a 2-holed torus "looks like"

**Lecture 22 Alex A.**
Explain what a knot is, and what it means for two knots to be equivalent. Explain exercise 6.1 and deduce Proposition 6.1. Briefly describe the torus knots and outline the proof of Prop 6.2. Do not worry about showing that these fundamental groups are non-isomorphic |
| 15    | Covering Spaces, Path Lifting Lemma, Homotopy Lifting Lemma | **Lecture 23 Alex F.**
Define the notion of a covering space. Select examples from the text that best illustrate the subtleties of this definition

**Lecture 24 Jacob**
Prove the very important path lifting and homotopy lifting lemmas (lemmas 3.1, 3.2, 3.3). If you have time, discuss lemma 3.4 |
<p>| 16    | Fundamental Group of a Covering Space, Lifting of | <strong>Lecture 25 Daniel</strong> |</p>
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| 04/07 | Arbitrary Maps to a Covering Space | Deduce theorem 4.1 and 4.2. Select examples from section 2 to work out instances of these theorems.  
*Lecture 26 Grace*  
Prove Theorem 5.1. How does this specialize to prove lemmas 3.1 and 3.3? This might be a short lecture, but I don't want to tack on another section. |
| 04/09 | Homomorphisms and Isomorphisms of Covering Spaces, Action of the Fundamental Group on Fibers of Covering Spaces | Discuss homomorphisms and isomorphisms of covering spaces - consider discussing exercise 6.2  
*Lecture 27 Grace*  
Explain the action of the fundamental group on the fibers of a covering space, and identify the "isotropy" of this action. Prove theorem 7.2, which identifies the group of "Deck transformations"  
*Lecture 28 Harrison* |
| 04/14 | Regular Covering Spaces and Quotient Spaces, Borsuk-Ulam Theorem for the 2-sphere | The most important thing here is to show that the covering space may be regarded as a quotient of a space by a suitably "discrete" group action. There are two counterexamples at the end of the chapter exhibiting some bizarre phenomena - they are rather detailed, but you can touch on the main point of these counter examples.  
*Lecture 29 Nerses*  
*Nothing builds upon this lecture. You can do it out of Hatcher (p. 32) or Munkres if you prefer.*  
The Borsuk-Ulam theorem is one of those wonderful theorems that we now have the technology to prove. This section is short - you could also do the exercises. |
| 04/16 | The Existence Theorem for Covering Spaces, Induced Covering Space over a Subspace | We've talked about the properties of covering spaces, but do these fabulous beasts always exist? The answer is - yes - provided the space is "semi-locally simply connected"  
*Lecture 31 Grace*  
For us a covering space has always been path connected. If you restrict a covering to a subspace, it will be a covering, except for the fact that it might not be path connected. Prop 11.2 gives a nice criterion. Work through some examples. |
| 04/23 | Final Project Draft due. Lecturer's Choice  
Graphs, Trees, Fundamental Group of a Graph | Grayed out text is from original syllabus. You may use it if you wish.  
*Lecture 33 Jacob*  
Lecturer's Choice: Choose some material from a book that looks interesting to you. Photocopy the pages you plan to cover, and circle the theorems, proofs and examples you plan to include. If material is from Hatcher or Massey, photocopy is unnecessary; just email me the section / prop / thm #’s. Submit this to me for approval (slide under my ofc door or email) at |
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| 21 T 04/28 | Lecturer's Choice | least 5 days before your lecture. These sections (Chapter 6, sections 2-3) contain many definitions and concepts concerning graphs. Explain these, and draw a lot of pictures to illustrate them.  
*Lecture 34 Manuel*  
Lecturer's Choice : See Lecture 33 for instructions. Define the notion of a tree, explain why trees are contractible, and explain the notion of maximal trees. Show that the fundamental group of a graph is free, and if there is time, discuss the number of generators. We already touched on some of this in our discussions, but it is worthwhile to prove this in detail. |
| 21 T 04/28 | Euler Characteristic and Coverings of Graphs, Generators of Subgroups of Free Groups | *Grayed out text below is from original syllabus. You may use if you wish.*  
*Lecture 35 Gabriel*  
Lecturer's Choice: See Lecture 33 for instructions. Discuss the relationship between Euler characteristic and the fundamental group of a graph. Discuss what the coverings of a graph look like. Prove the all important theorem 7.2  
*Lecture 36 Alex F.*  
Lecturer’s Choice: See Lecture 33 for instructions. Explain how to produce generators of subgroups of free groups. Discuss this remarkable theory in the context of many examples. |
| 22 R 04/30 | Daniel and Alex F. | |
| 23 T 05/05 | Presentations of Final Projects | |
| 24 R 05/07 | Final Project due. | Presentations of Final Projects |
| 25 T 05/12 | Presentations of Final Projects | |
| 26 R 05/14 | Presentations of Final Projects | |