This is a 65-minute exam. The actual exam will be a little bit shorter.

1. (27 points) Evaluate:
   (a) \( \int \frac{dx}{x^3 + x^2} \).
   (b) \( \int e^{x^2} \frac{dx}{x \ln^2 x} \).
   (c) \( \int \frac{x^3 dx}{\sqrt{1-x^2}} \).

2. (15 points) Prove that \( \lim_{x \to +\infty} f(x) = 3 \) if and only if \( \lim_{t \to 0^+} f(1/t) = 3 \).

3. (23 points) Prove that a sequence can not converge to two different limits.

4. (12 points) There is a positive integer \( m \) such that
   \[
   \lim_{x \to 0} \frac{\sin(2x^3) - 2x^3}{x^m}
   \]
   is finite and nonzero. What is \( m \), and what is the limit \( L \)?

5. (27 points) Evaluate:
   (a) \( \lim_{x \to 0} \frac{\sin^2(ax)}{1 - \cos(bx)} \).
   (b) \( \lim_{x \to 1} \left( \frac{1}{\ln x} - \frac{1}{x - 1} \right) \).
   (c) \( \lim_{x \to 0^+} \frac{e^{-1/x}}{x} \).

6. (26 points) If \( f(x+y) = f(x)f(y) \) for all \( x \) and \( y \) and if \( f(x) = 1+ xg(x) \), where \( g : \mathbb{R} \to \mathbb{R} \) such that \( \lim_{x \to 0} g(x) = 1 \), prove that
   (a) \( f'(x) = f(x) \) for every \( x \),
   (b) \( f(x) = e^x \). [Hint: Let \( h(x) = e^{-x}f(x) \) and take its derivative.]