You may not use any books, notes, or electronic devices on the exam. Showing work will result in more partial credit. You may not use any calculus theorems or techniques that have not been covered in this course. Good Luck!

1. (16 pts.) State the theorems below. Be sure to state the hypotheses.
   
   (a) Mean value theorem for derivatives.

   (b) First fundamental theorem of calculus.

2. (40 pts.) True/False. For false statements, no credit without a valid counter-example. (If your counter-example involves a function, be sure to specify the domain and range.) If a statement is true, you do not have to justify your answer. True statements are not worth as many points as false statements.

   (a) If $f$ is uniformly continuous on a set $S \subseteq \mathbb{R}$, then $f$ is continuous at every $x \in S$. 
(b) ___ If \( f \) is continuous at every \( x \in S \subseteq \mathbb{R} \), then \( f \) is uniformly continuous on \( S \).

(c) ___ Suppose that \( f \) is a function that is continuous on some set \( S \subseteq \mathbb{R} \). Then \( f \) attains its maximum and minimum on \( S \). That is, there exist \( x_m, x_M \in S \) such that
\[
f(x_m) \leq f(x) \leq f(x_M) \quad \text{for all} \quad x \in S.
\]

(d) ___ If a function is continuous at \( c \in \mathbb{R} \) it must also be differentiable at \( c \).

(e) ___ Every strictly increasing function from [0, 1] to [0, 1] has an inverse. [Recall that a function has an inverse if it is both injective and surjective.]

(f) ___ If a function is differentiable at \( c \in \mathbb{R} \) it must also be continuous at \( c \).
(g) If a function $f : [a, b] \rightarrow \mathbb{R}$ is Lipschitz, then it is uniformly continuous. [Recall that $f$ is Lipschitz if there exists some $k \in \mathbb{R}^+$ such that for all $x, y \in [a, b]$, we have the inequality $|f(x) - f(y)| < k|x - y|$.]

(h) If $f$ is continuous on $[a, b]$ then $\int_a^b f(x) \, dx$ exists.

(i) Let $f$ be integrable on $[a, b]$ and define $A(x) = \int_a^x f(t) \, dt$. Then $A(x)$ is continuous.

(j) Suppose $f$ and $A(x)$ are as above. Then $A(x)$ is differentiable on $[a, b]$. 
3. **(40 pts.)** In this problem, you cannot receive full credit unless you show your work. Suppose \( f \) is a function defined and continuous for all \( x \in \mathbb{R} \), and that

\[
\begin{align*}
 f(1) &= 2 \quad \text{and} \quad f(2) = 3 \quad \text{and} \quad f(3) = 7; \\
 f'(1) &= 5 \quad \text{and} \quad f'(2) = 7 \quad \text{and} \quad f'(3) = 2; \\
 f''(1) &= 1 \quad \text{and} \quad f''(2) = 3 \quad \text{and} \quad f''(3) = 1.
\end{align*}
\]

Let \( h(x) = f(f(x)) \); compute the following values.

(a) \( h'(1) \).

Write your answer here: \( h'(1) = \) ______

(b) \( h''(1) \).

Write your answer here: \( h''(1) = \) ______

Suppose that \( f \) is invertible and let \( g(x) \) be its inverse. Compute:

(c) \( g(3) \).

Write your answer here: \( g(3) = \) ______

(d) \( g'(3) \).

Write your answer here: \( g'(3) = \) ______
[Table from previous page:]

\[
\begin{align*}
f(1) &= 2 \quad \text{and} \quad f(2) = 3 \quad \text{and} \quad f(3) = 7; \\
f'(1) &= 5 \quad \text{and} \quad f'(2) = 7 \quad \text{and} \quad f'(3) = 2; \\
f''(1) &= 1 \quad \text{and} \quad f''(2) = 3 \quad \text{and} \quad f''(3) = 1. \\
\end{align*}
\]

In the next part, if you use any major theorems or properties, cite them.

Let \( A(x) = \int_{x}^{x+2} [f(t)]^2 \, dt \).

(e) Then \( A'(1) = \)_________