Problem set 1

This problem set is due in class on Feb 18th, 2015.

1. Exercise 1-2 of the bipartite matching notes.

2. Exercise 1-4 of the bipartite matching notes.

3. Exercise 1-5 of the bipartite matching notes.

4. (More difficult.) Let $S = \{1, 2, \cdots, n\}$. Let $A_k$ be the set of all subsets of $S$ of cardinality $k$ (thus $|A_k| = \binom{n}{k}$). Let $k < \frac{n}{2}$. Consider the graph $G_k$ with bipartition $A_k$ and $A_{k+1}$, and with $E = \{(a, b) | a \in A_k, b \in A_{k+1} \text{ and } a \subset b\}$.

   (a) Prove that the maximum matching in $G_k$ has size $A_k$ (remember $k < n/2$).

   (b) Prove Sperner’s lemma. The maximum number of subsets of $S$ such that no subset is contained into another is $\binom{n}{\lfloor n/2 \rfloor}$.