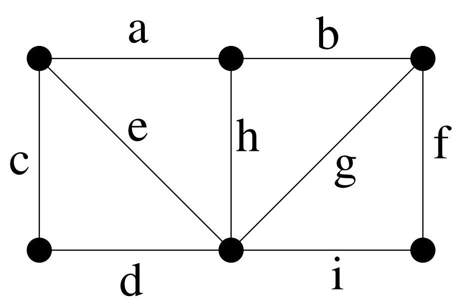
Massachusetts Institute of Technology
Michel X. Goemans

18.433: Combinatorial Optimization
April 18th, 2013

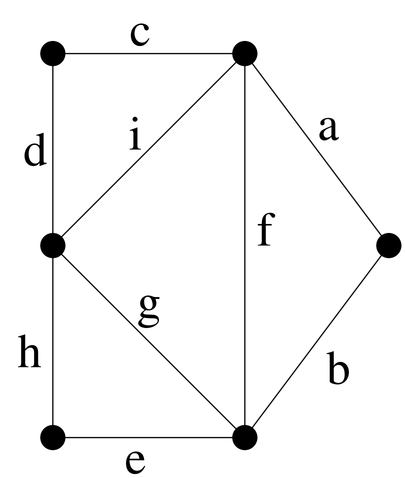
Problem set 4

This problem set is due in class on April 25th, 2013.

1. We are given the following two graphs G_1 and G_2 with edge set $E = \{a, b, c, d, e, f, g, h, i\}$.



G_1



G_2

   Observe that $S = \{a, b, c, d\}$ is a forest in both G_1 and in G_2, so it is independent in $M_1 = M(G_1)$ and $M_2 = M(G_2)$. Construct the exchange graph corresponding to S, and indicate which elements are sources and sinks. Using the exchange graph, find a larger set of elements which is acyclic in both G_1 and in G_2.

2. Consider a graph $G = (V, E)$ with $|E| = 2(|V| - 1)$ and suppose the edges are partitioned into $|V| - 1$ blue edges (in B) and $|V| - 1$ red edges (in R). Suppose furthermore that G is the union of two edge-disjoint spanning trees (with no restriction on the colors of the edges). Show that G contains a tree with at most $\lceil \frac{|V|-1}{2} \rceil$ blue edges and with at most $\lceil \frac{|V|-1}{2} \rceil$ red edges. (Hint: think polytopes...)

3. Exercise 5-1 from the matroid intersection notes.

4. Exercise 5-4 from the matroid intersection notes.

5. For the graduate students: Consider any matroid $M = (E, \mathcal{I})$.

   Given two bases (maximal independent sets) B_1 and B_2 of M, we know that for every $e \in B_2 \setminus B_1$ we can find $f \in B_1 \setminus B_2$ such that $B_1 + e - f \in \mathcal{I}$. Prove that given $e \in B_2 \setminus B_1$, we can find $f \in B_1 \setminus B_2$ such that $B_1 + e - f \in \mathcal{I}$ **and** $B_2 - e + f \in \mathcal{I}$.