Problem set 2

This problem set is due in class on March 1st, 2011.

1. Exercise 2-6 from the notes on non-bipartite matchings.

2. A graph $G = (V, E)$ is said to be *factor-critical* if, for all $v \in V$, we have that $G \setminus \{v\}$ contains a perfect matching. In parts (a) and (b) below, G is a factor-critical graph.

   (a) Let U be *any* minimizer in the Tutte-Berge formula for G. Prove that $U = \emptyset$. (Hint: see Exercise 2-3 from the notes.)

   (b) Deduce that when Edmonds algorithm terminates the final graph (obtained from G by shrinking flowers) must be a single vertex.

   (c) Given a graph $H = (V, E)$, an *ear* is a path $v_0 - v_1 - v_2 - \cdots - v_k$ whose endpoints (v_0 and v_k) are in V and whose internal vertices (v_i for $1 \leq i \leq k-1$) are not in V. We allow that v_0 be equal to v_k, in which case the path would reduce to a cycle. Adding the ear to H creates a new graph on $V \cup \{v_1, \cdots, v_{k-1}\}$. The trivial case when $k = 1$ (a 'trivial' ear) simply means adding an edge to H. An ear is called *odd* if k is odd, and even otherwise; for example, a trivial ear is odd.

      i. Let G be a graph that can be constructed by starting from an odd cycle and repeatedly adding odd ears. Prove that G is factor-critical.

      ii. Prove the converse that any factor-critical graph can be built by starting from an odd cycle and repeatedly adding odd ears.

3. Exercise 3-1 from the notes on polyhedral theory.

4. Exercise 3-2 from the notes on polyhedral theory.