1. Consider the following tableau during the execution of the simplex algorithm with some constants replaced by placeholders $q, r, s, t$ (the first row is the $c$ vector, the last column is the $b$ vector):

$$
\begin{array}{ccccc|c}
-z & x_1 & x_2 & x_3 & x_4 & x_5 \\
1 & 0 & r & 0 & 0 & t & -12 \\
0 & 1 & 0 & 1 & -2 & 1 \\
0 & -3 & 1 & 0 & 0 & 5 \\
1 & q & 0 & 0 & s & 9
\end{array}
$$

(a) What is the basic feasible solution corresponding to this tableau?

$$x = (x_1, x_2, x_3, x_4, x_5) =$$

What conditions must these parameters $q, r, s, t$ satisfy

(b) for the current bfs to be optimum?

(c) to allow $x_2$ to enter the basis and to allow $x_1$ to leave it in the same pivoting step?

(d) so that the current tableau certifies that the linear program is unbounded?

(e) Now, assume that $r = 3, t = -1, q = 2$ and $s = 3$.

$$
\begin{array}{ccccc|c}
-z & x_1 & x_2 & x_3 & x_4 & x_5 \\
1 & 0 & 3 & 0 & 0 & -1 & -12 \\
0 & 1 & 0 & 1 & -2 & 1 \\
0 & -3 & 1 & 0 & 0 & 5 \\
1 & 2 & 0 & 0 & 3 & 9
\end{array}
$$

Do one pivoting step. Indicate which variable enters the basis and which one leaves it.

$$
\begin{array}{ccccc|c}
-z & x_1 & x_2 & x_3 & x_4 & x_5 \\
1 & 1 & x_2 & x_3 & x_4 & x_5 \\
\end{array}
$$

(f) What is the resulting bfs and is it optimal?
2. Consider the following network and a flow $x$ in it. On each arc (directed edge), the first value indicates the flow on it, and the second indicates its capacity.

(a) Draw the residual graph corresponding to this flow $x$ and indicate on each arc its residual capacity (do not forget to indicate the direction of every arc).

(b) Is there an augmenting path? **YES** / **NO**.

- If **YES**, show an augmenting path and a flow of greater value. Is the resulting flow maximum? Justify
- If **NO**, exhibit an $s-t$ cut of equal capacity and explain in one sentence how you found it.

3. For each statement below, state whether it is True or False AND give a (brief) justification of your answer.

(a) The number of comparisons required to merge two sorted lists, one with \( n_1 \) keys and the other with \( n_2 \) keys, is at least \( \log_2 \binom{n_1 + n_2}{n_1} \).

True / False.

(b) The largest \( k \) elements of an unsorted array of \( n \) elements can be extracted (in sorted order) using at most \( c(n + k \log n) \) comparisons for some constant \( c \).

True / False.

(c) In QUICKSORT, suppose one selects the pivot (used to partition the list) by running a median-finding algorithm that requires only \( cn \) comparisons (for some \( c > 0 \)). Then there exists arbitrarily large \( n \) such that the total number of comparisons for sorting a list of \( n \) inputs by this algorithm can be (on some inputs) at least \( c'n^2 \) for some constant \( c' > 0 \).

True / False.

(d) If the number of comparisons of an algorithm on inputs of size \( n \) satisfies \( T(n) \leq T(n/2) + T(n/3) \) (or, more precisely, \( T(n) \leq T(\lfloor n/2 \rfloor) + T(\lfloor n/3 \rfloor) \)) and say \( T(i) = i \) for \( i \leq 3 \) then \( T(n) \leq cn \) for some constant \( c \).

True / False.

(e) One can derive a sorting network based on INSERTIONSORT with \( \binom{n}{2} \) comparators.

True / False.

4. What is the multiplicative inverse of 39 modulo 140?