Practice Quiz 2

1. Consider the following tableau during the execution of the simplex algorithm with some constants replaced by placeholders q, r, s, t (the first row is the c vector, the last column is the b vector):

$-z$	x_1	x_2	x_3	x_4	x_5	
1	0	r	0	0	t	-12
	0	1	0	1	-2	1
	0	-3	1	0	0	5
	1	q	0	0	s	9

   (a) What is the basic feasible solution corresponding to this tableau?

$$x = (x_1, x_2, x_3, x_4, x_5) =$$

   What conditions must these parameters q, r, s, t satisfy

   (b) for the current bfs to be optimum?

   (c) to allow x_2 to enter the basis and to allow x_1 to leave it in the same pivoting step?

   (d) so that the current tableau certifies that the linear program is unbounded?

   (e) Now, assume that $r = 3$, $t = -1$, $q = 2$ and $s = 3$.

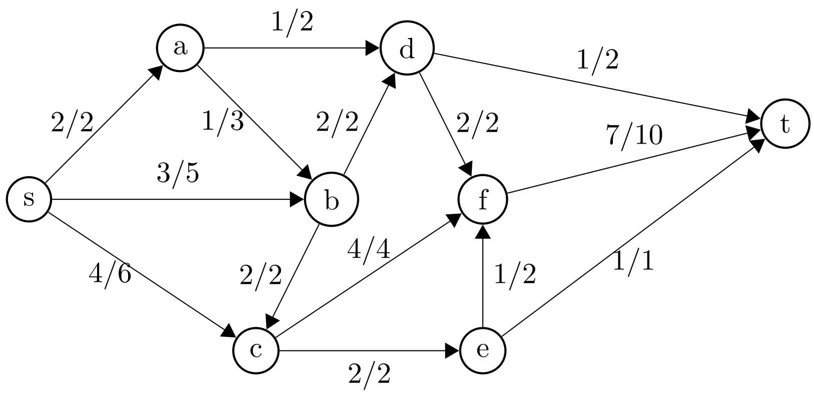
$-z$	x_1	x_2	x_3	x_4	x_5	
1	0	3	0	0	-1	-12
	0	1	0	1	-2	1
	0	-3	1	0	0	5
	1	2	0	0	3	9

   Do one pivoting step. Indicate which variable enters the basis and which one leaves it.

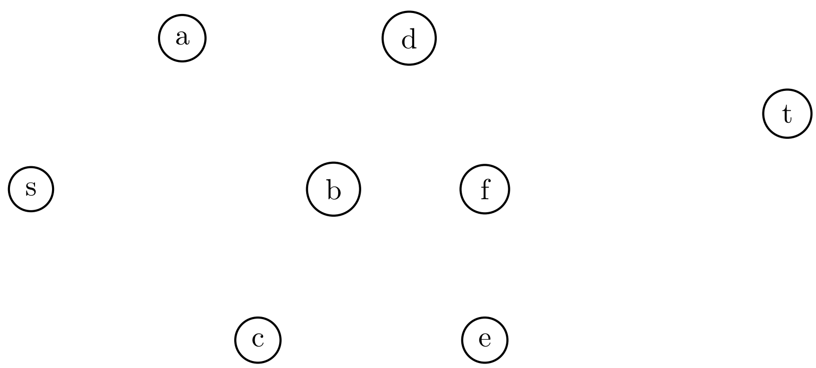
$-z$	x_1	x_2	x_3	x_4	x_5	
1						

   (f) What is the resulting bfs and is it optimal?

2. Consider the following network and a flow x in it. On each arc (directed edge), the first value indicates the flow on it, and the second indicates its capacity.

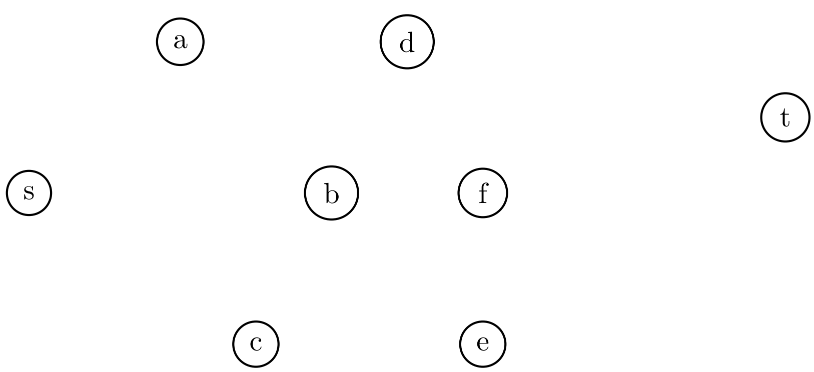


(a) Draw the residual graph corresponding to this flow x and indicate on each arc its residual capacity (do not forget to indicate the direction of every arc).



(b) Is there an augmenting path? **YES** / **NO**.

- If **YES**, show an augmenting path and a flow of greater value. Is the resulting flow maximum? Justify
- If **NO**, exhibit an $s-t$ cut of equal capacity and explain in one sentence how you found it.



3. For each statement below, state whether it is True or False **AND** give a (brief) justification of your answer.

   (a) The number of comparisons required to merge two sorted lists, one with n_1 keys and the other with n_2 keys, is at least $\log_2 \binom{n_1+n_2}{n_1}$.
   True / **False**.

   (b) The largest k elements of an unsorted array of n elements can be extracted (in sorted order) using at most $c(n + k \log n)$ comparisons for some constant c.
   True / **False**.

   (c) In QuickSort, suppose one selects the pivot (used to partition the list) by running a median-finding algorithm that requires only cn comparisons (for some $c > 0$). Then there exists arbitrarily large n such that the total number of comparisons for sorting a list of n inputs by this algorithm can be (on some inputs) at least $c'n^2$ for some constant $c' > 0$.
   True / **False**.

   (d) If the number of comparisons of an algorithm on inputs of size n satisfies $T(n) \leq T(n/2) + T(n/3)$ (or, more precisely, $T(n) \leq T(\lfloor n/2 \rfloor) + T(\lfloor n/3 \rfloor)$ and say $T(i) = i$ for $i \leq 3$) then $T(n) \leq cn$ for some constant c .
   True / **False**.

   (e) One can derive a sorting network based on InsertionSort with $\binom{n}{2}$ comparators.
   True / **False**.

4. What is the multiplicative inverse of 39 modulo 140?