By a suitably souped-up version of this construction, we map sheaves on $X \times X$ to $A_{\infty}$-bimodules over $A$. In particular, $(n+1 = \dim_{\mathbb{C}} X)$
\[
\text{Hom}_{X \times X}^0 (\mathcal{O}_X \otimes X_0 \otimes \mathcal{O}_X) \rightarrow H^{n+1} (\text{hom}_{\text{bimod}(A)} (\hat{A}, \hat{A}))
\]
\[
H^0 (X; K^y_X)
\]
\[
\hat{A} \quad \hat{A}
\]
\[
\delta : \hat{A} \rightarrow \hat{A} [-n-1]
\]
\[
\{ \delta \} : \hat{A} \rightarrow \hat{A} [-n-1]
\]
\[
\mathcal{B} \quad \mathcal{D}^b (X)
\]
\[
\text{sections of } K^y_X \text{ (for } \mathbb{C}P^2, O(1))
\]
\[
\text{(in our case, this section is } s = z_0 z_1 z_2 \text{)}
\]
\[
\mathcal{B} \text{ as an } A_{\infty} \text{-algebra}
\]
\[
\text{(describes } \rho^{-1}(0) = M)
\]
\[
\mathcal{B} \text{ as an } A_{\infty} \text{-algebra}
\]
\[
\text{(describes } \rho^{-1}(0) = M)
\]
\[
\text{Local Mirror Symmetry}
\]
\[
C = \mathbb{C} (y_1, y_2, x_1, x_2) \in \mathbb{C}^2 \times \mathbb{E}^3
\]
\[
y_1^2 + y_2^2 = \rho(x_1, x_2)
\]
\[
\text{where } E = (\mathbb{C}^2)^2, \rho(x_1, x_2) = x_1 + x_2 + \frac{1}{x_1 x_2}
\]
\[
X = \mathbb{C}P^2, \quad K = \text{Tot} (\mathcal{K}_X \rightarrow X)
\]
\[
(\text{note } K \text{ is a resolution of } \mathbb{C}^3 / (\mathbb{Z}/3))
\]
We are interested in sheaves on $K$ supported on the zero section $X \subset K$.

In particular, we can look at

$$\bigoplus_{i, j} \text{Hom}_K^* \left( E_i, E_j \right) \cong \bigoplus_{i, j} \text{Hom}_X^* \left( E_i, E_j \right) \oplus \text{Hom}_X^{n + 2 - *}(E_j, E_i)^*$$

(trivial extension by the dual bimodule).

Why? $\text{Hom}_K^* \left( E_i, E_j \right) \cong K \to X$

$$= \text{Hom}_K^* \left( O_X \otimes \pi^* E_i, \pi^* E_j \right)$$

$$= \text{Hom}_K^* \left( \pi^* \left( K \otimes \pi^* E_i \right), \pi^* E_j \right)$$

$$= \text{Hom}_K^* \left( \pi^* \left( K \otimes \pi^* E_i \right), \pi^* E_j \right)$$

$$= \text{Hom}_X^* \left( \left( K \otimes \pi^* E_i \right)[1] \otimes \pi^* E_j \right), \text{several ways}.$$
**Theorem:** (Ballard) Let $D$ be the differential graded algebra underlying $\bigoplus \text{Hom}_K^*(E_i, E_j)$. Then the dga underlying $\bigoplus \text{Hom}_K^*(E_i, E_j)$ is quasi-isomorphic to the trivial extension $D \otimes D^*[\dim_K C]$. 

A Lefschetz fibration $p : E \to C$ gives rise to a pair of $A_n$-algebras $(\hat{A}, B)$, $A \subset B$. 

**Def:** The suspension of $p$ is $p^\sigma : (C \times E) \to C$, $p^\sigma(x, y) = y^2 + p(x)$ 

$E^\sigma$ (choice of $V_i$ determines choice of vanishing cycles on $E^\sigma$). $(A^\sigma, B^\sigma)$ associated to $p^\sigma$, then $\hat{A}^\sigma = \hat{A}$, but $B^\sigma \neq B$. 

**Theorem:** $B^\sigma$ is the trivial extension of $\hat{A}$ by $\hat{A} \otimes D^*[\dim_K C - 1]$. 

**Cor:** (also holds for all toric del Pezzo surfaces) Let $D^b_X(K)$ be the derived category of coh. sheaves $\text{sh}_X$ w/ cohomology supported on $X$. Let $F(C)$ be the Fukaya category of the mirror $C$. Then, there is a full embedding $D^b_X(K) \hookrightarrow D^b(F)$. 
