\[ \text{Hom}_A^*(E_i, A) \xrightarrow{\text{claim}} \lim_{\rightarrow P} \text{Hom}_X^*(E_i \otimes \mathcal{O}_X \otimes P, X) \quad \text{direct limit depends on } s \in H^0_X(K_X^{-1}) \]

\[ = \lim_{\rightarrow P} H^X_X(E_i \otimes \mathcal{O}_X \otimes P, X \setminus s^{-1}(0)) = A \]

\[ X = \mathbb{C}P^2 \]
\[ S = 2 \mathbb{Z} \mathbb{Z} \subseteq H^0(X, \mathcal{O}(3)) \]
\[ A = (\mathbb{C}^*)^2 \]

\[ \text{Claim: } \text{Hom}_A^*(\mathcal{O}_A, \mathcal{O}_A) = \mathbb{C}[t_{0}, t_{2}]^+ \]

\[ \text{Application: Look at } \text{HW}, \text{ SH.} \]

\[ \text{HW is a module } \text{over } SH \]

\[ \text{Assume } SH = 0. \text{ (All critical steps, e.g., lines).} \]

\[ \text{Limit ring } \Rightarrow \text{HW} = 0. \]

\[ \Rightarrow \text{lower hands on Reeb cord.} \]

\[ \text{Bla-bla are top to } \text{HW, top of } C_s^{-1} \text{ path.} \]

\[ \text{By P.D. LES, etc.} \]

\[ \text{Example: } D < S^{2\pi} \text{ (Legendrian bands exact } \mathcal{L} \text{, } L \text{?)} \]

\[ B^{2\pi} \Rightarrow \text{Then generally } \Rightarrow \text{Reeb chords for } D \geq \dim (\mathcal{H}_C) \geq \frac{1}{2} \dim (\text{HW} \mathcal{A}) \]
Cyclic A∞ algebra:  
\( V \) f.d. vector space, 
\( \langle \cdot, \cdot \rangle : V \otimes V \to \mathbb{C} \) non-degenerate  
\( \exists y^d \) such that \( \langle y^d(\cdot), \cdot \rangle \) is cyclically symmetric. 
(For example, non-compact CY with \( c_1 = 0 \)).

Lev: (K5) Given an A∞ structure on \( V \), and a cyclic co-chain \( \alpha \in \text{CC}^{n-2}(V, V) \) which induces a non-degenerate form on \( H^1(V) \). Can make a quasi-isomorphic cyclic A∞ structure.

How do we get \( \alpha \in \text{CC}^{n-2}(V, V) \)? This is a linear problem.

In "Goettsch's" book on Cyclic Homology.

Connes-Segal?

Why \( d \alpha \)? We have no idea what Connes' complex is.

\( C-Y \leftrightarrow \text{CY} \)

More like \( C-Y \leftrightarrow \text{compactness?} \)

e.g.\( \text{cochains on an } \text{cpt.-mfld} \) is always \( C-Y \)?

Why is making an A∞ structure cyclic a good thing, to do?

Paul: Need these for open GW invariants. Lots then only do connection forms

(formal Chern-Simons theory?) make sense.

Also to apply Costello to fun this into a TCFT, stats CY, cyclic A∞ structure.

Hopkins-Lurie on stable op. rely on some non-nilpot generalization of cyclic A∞ struc.
Wrapped Fock scheme also has a co-product;

to recover this might also need cyclic structure.

\[ \frac{B}{A} = A \] is a weak version of this analogy.

Closed string A-model LG is \[ \text{Invis-
\text{-Fan-Phan} on } \]
Solution to Witten's equation.

Wronskian singularities is another regularity.

\[ X \xleftarrow{\phi} \phi \]

D hypersurface \[ \langle D \rangle = c_1(X) \]

W mirror to \( X \xrightarrow{\psi} \phi \) (using Seg tori \( C \times \langle D \rangle \)).

W counts hol. discs \[ \mathbb{C}^2 \xrightarrow{\psi} \]
\[ \mathbb{C}^* \]

\[ M = \mathbb{C}^* \]
\[ W = \mathbb{C} \mathbb{C} \frac{z}{z} \]

Paul: Can you do \( \mathbb{C}^2 \)/divisor have a double point?

Denis: slags become new spec, don't know how to del.

\[ \mathbb{C}^2 \rightleftarrows \mathbb{C}^2 \]

\[ \Omega = \frac{dz_1 \wedge dz_2}{z_1 z_2} \]

\[ W = z_1 z_2 + e^{-c} \frac{z_1 z_2}{z_1 z_2} \]

\[ \operatorname{fib} \]

\[ \mathbb{C}^2 \setminus \text{slas} \to \infty \]
\[ \mathbb{C}^2 \setminus \text{slas} \to 0 \]
\[ S = z_0 (z_1 z_2 - \varepsilon z_0^2) \]

\[ \varepsilon S = z_0 (z_1 z_2 - \varepsilon z_0^2) \]

\[ S = z_0 (z_1 z_2 - \varepsilon z_0^2) \]

\[ \frac{z_1^2 - z_2^2}{z_1^2 + z_2^2} = c_1 \]

\[ z_1 z_2 - \varepsilon z_0^2 = c_2 \]
What if we went all the way to smooth cubic?

\[ D = \text{smooth cubic in } \mathbb{P}^2. \]

\[ \mathbb{P}^2 \setminus D \text{ should carry a \textbf{St}} \quad T^2\text{-fibred} \]

\[ w/ 3 \text{ singular fibers} \]

What is the structure of fibration?

Look at \( W = 1 \), notice

\[ \text{Claim: } \exists \, \nu = 2 \text{ is mirror to } D. \]
Juan: posing to local $P^2$ is like stabilizing twice. Is there a mirror to stabilizing once?

Denis: $E = T^2 \quad S^2 = \emptyset \quad \text{base if } SYZ = 0 \quad \text{2:2 glue adjacent branches at 4 pts.} \quad \text{base } = \text{I}$

On $(CP^1, \alpha, \Omega)$

$$\Omega = \frac{dz}{\sqrt{(z-a)(z-b)(z-c)}}$$

quadratic of flat col. fan, 1 elliptic core

use to build a mirror

mirror of $E$

$$\Gamma^1_2$$

mirror of $CP^1$

branches at fibre, fibre has some how mirror is a fibration corresponding to (branched) point

$(x, 1^+) \quad \ni \quad 1 - 2K_x$

Actual mirror to $CP^1$ is annulus in $C^*$:

$$W = z + \frac{a}{z}$$

$|z| < 1$

$|a| < 1$

We can take large vol. limit

mirror of $K_x$, fibre $D^2$ double cone along $2D^2$.

Can think of $F(x, y)$ as $\text{Fun}(F(K), F(K))$

of sheaves in

e.g. $P^1 \times P^1$. these are free. these are exotic. these are

Pauks. monodromy lattices in $T^*S^2$ are either compatible or not with $S^2$-polychronous torus

look at $F(x, y)$ as sheaves, seems like nothing except $S^2$-polychronous torus.
\[ S^2 \times S^2 \text{ with } \xi \text{ as above, by Avno, } \alpha = 0 \text{ or } \gamma \text{ as before, have been written.} \\
\text{(and similar splittings, etc.).) }

**Claim:** (2 analytic change of vars. which matches \( W \) across walls. \\
(How to define \( \pi \) for \( \sigma = 0 \) classes?)

Almost always false that set of walls is locally finite. \\
 Might have a dense set of walls, with many jumps. \\
Analysis is hard to do, might have multi covers of \( \sigma \). \\
(Deep within FODO).

**B) to examples:** \( \mathbb{C}P^2, D = \{ z_1 z_2 = z_3 \} \cup \{ 0 \leq z_4 \} \).

\[ \mathcal{Z} = \frac{dz_1 dz_2}{z_2 z_3 - \overline{z_2} z_3} \]

\[ \mathbb{C}P^2 \setminus D \text{ has a \( \mathbb{S}^3 \text{-fiberation } \mathbb{S}^4 \to \mathbb{R}^3. \)} \]

\[ (z_1, z_2) \mapsto (z_1 e^{i\theta}, z_2 e^{-i\theta}) \]

\[ TV_{\epsilon, \lambda} = \{ z_1 z_2 - \epsilon = 0 \} \]

How to prove this?
Lefschetz fibration - line singularity at origin

If \( r = 3 \), then \( \lambda = 0 \).

\[ T_{131,0} = \]

Exercise: They're slay.  \( \sigma \) counts \( S^1 \) action.

[Summary: \( T(T_{r,2}) \oplus V = (r_1z_1 - r_2z_2)^7 \)]

\[ \iota_V^\ast \omega = d\phi^* \, s_0 \quad \phi^* \, S_0 \Rightarrow \log r \cdot \]

\[ \iota_V^\ast \Omega = \iota d \log (r_1z_2 - r_2z_1)^5 \quad \text{and} \quad r_1z_2 - r_2z_1 = r \Rightarrow \text{Special} \]

[Mohamed: which almost?  Neither: \( \lambda = 0 \) sympl, \( \log r = \text{gph}\phi \).

If \( r = 3 \),

Well-crossing: \( \lambda \neq 0 \), then above \( 0 \), looks like:

Two cases:

\( r = 1 \) - can deform \( T_{r,2} \), namely to product \( r_1z_1 \), (bubbling - (trivial circle till center) at origin)

\( 3 = 0 \) -

\( T_{r,2} \) has 3 families of \( y = 2 \) discs, same as before

\[ W = r_1z_1 + e^{-A} \sqrt{z_2 - \delta_0 - r_1 - z_2} \]

- multi section over complement of \( \mathbb{P}^2 \)
2=2 case = Chekanov torus

1 family of \( n = 2 \) discs inside \( \mathbb{R}^2 \).
3 families through line at \( 0 \).
[Chekanov-Schlenker, Polterovich - B -]

In this case:
\[ n = 1. \]
each of \( n_{\text{cop1}} \neq n_{\text{cop2}} = 1. \)
\[ n_{\text{cop1}} = n_{\text{cop2}} = 2. \]

\[ W = \frac{u + 2e^{-A}}{u^2 + e^{-A} + e^{-A}v} + \frac{e^{-A}}{u^2v + e^{-A}v} \]

\[ \Rightarrow \left( u + \frac{e^{-A}}{u^2v + e^{-A}v} \right) \]

\[ \Rightarrow \left( u^2 + 2v \right) + \frac{e^{-A}}{2v} \]

\[ \Rightarrow u = z_1 + z_2 \]
\[ v = \frac{z_1z_2}{2} \]

Gluing:
\( \beta \leftarrow \beta_1 \) or \( \beta_2 \) depending on \( \text{sign } \theta \).
\[ \kappa \leftarrow \beta_2 - \beta_1 \), guess \( \kappa \).

\[ \beta \leftrightarrow \beta_1 \]
\[ u = z_1 \]
\[ r \]
\[ u = z_2 \]

\[ \kappa \]
\[ \beta_2 \]

Amount for bubbling, tells us we'll possibly pick up exceptional disc & glue us two discs.

So the answer is obtained by gluing these two curves n (the following charge of course).