1) Examples
2) Birationl Geometry
3) Conic Bundles

(1) $(2,3)$ curve $\subset \mathbb{P}^1 \times \mathbb{P}^1$

- Coordinates: $y, x_1, x_2, x_3, x_4$

- Using Hori-Vafa, one can construct mirror as:

\[
\begin{align*}
    x_1 x_2 &= u^2 \\
    x_0 x_3 x_4 &= u^3 \quad \text{and} \quad w = x_0 \quad \text{blow-up}
\end{align*}
\]

- Projected coordinate: $x_0$

- Desingularize this toric var, get a family of $K3$.

\[
y_i : \mathbb{P}^2 \to \mathbb{P}^2
\]

\[
y_0 = y_1 - y_2 - y_3
\]

- $F \in \text{D}^b\text{constr.}(\mathcal{O}_0)$, "equivariant"

- $\dim H^1(F) = 2$

- $\dim H^2(F) = 2$

- $\dim H^3(F) = 2$

- $\phi : Y_t \to Y_0$ def.

- $r : \mathbb{Z} y_t \to \mathbb{Z} y_0$

- Exact $\Delta$
Thin (cross, $K$): Let $X$ be a manifold of general type, $\dim X = 3$. Then $H^3(X) \cong H_1(F)\mathbb{C}_{-17}$.

It may be obtained another way:

Consider following conic bundle (toric conic bundle)

According to physicists, $x^2 + y = 0$ is a conic bundle.

$\mathbb{P}^1 \times \mathbb{P}^1$.

\[ u \psi = (1 + x + y)^2 \]

Mirror:

$X^3 \cup = z^2, z^2, z^3$

Gluing procedure:

2-dimensional Fano $X$

e.g. $X = 3$-dim. cubic. (using Hari-Uehara-Batyrev, Givental)

$xyuvw = (u + uvw)^2 \mathbb{P}^2 f = -x + y$

\[ X \times \left[ H^1(f) \cong \mathbb{C} \right] \]
$X$-smooth Fano of dim 3

$H^2_{\nu}(Y_t)$, $Y_t$ smooth fiber of $Z \to X$ and

$m$ monodromy acts on $H^2_{\nu}(Y_t)$ around $t = 0$.

Thus: Let $X$ be a smooth semisimple Fano

\[
\left\{ \begin{array}{c}
M \text{ is quasi-} \\
\text{semisimple but} \\
\text{not unipotent}
\end{array} \right\} \rightarrow \left\{ \begin{array}{c}
\text{not rational?}
\end{array} \right\}
\]