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18.996: Kontsevich's proof of Witten's conjecture and related topics

Witten's conjecture [2] predicted that some very natural intersection numbers on the moduli space of pointed curves are organized in such a way that their generating function satisfy infinitely many differential equations (the KdV hierarchy, or equivalently the Virasoro constraints). These intersection numbers are of particular interest because they often occur in enumerative geometry.

The conjecture was proved first by Kontsevich [1] using a variety of tools.

We will discuss Kontsevich's proof and use it to make ample digressions on the several topics, among which: stable Riemann surfaces and hyperbolic metrics; the moduli space of Riemann surfaces (as an orbifold) and Teichmüller space; the universal family and the tautological classes; the mapping class group; spines of hyperbolic surfaces with boundary and ribbon graphs; Jenkins-Strebel differentials and ribbon graphs; equivariant cellular model for the Teichmüller space and homology of the mapping class group; matrix integrals, Wick's lemma and enumeration of graphs; Sato's Grassmannian and solutions of the KP (and KdV) hierarchy.

Prerequisites are: a basic knowledge of Riemann surfaces, line bundles, uniformization theorem, basic algebraic topology and Chern classes.

Bibliography

[1] M. Kontsevich. Intersection theory on the moduli space of curves and the matrix Airy function. *Comm. Math. Phys.*, 147(1):1–23, 1992.

[2] E. Witten. Two-dimensional gravity and intersection theory on moduli space. In *Surveys in differential geometry (Cambridge, MA, 1990)*, pages 243–310. Lehigh Univ., Bethlehem, PA, 1991.