Erratum to *Erdős-Szekeres-type theorems for monotone paths and convex bodies*

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February 15, 2013

In Theorem 4.3, the proof is incorrect in the case $k = 4$, but works for larger k. In the case $k = 4$, we have instead the slightly weaker bound

$$N_4(q, 2n-1) > 2^{N_3(q,n)-1}. \tag{1}$$

To prove (1), we use the standard version of the stepping-up lemma (see, e.g., Chapter 4, Lemma 17 of [2]). The coloring is a variant of the coloring in the proof of Theorem 4.3. As in the proof of Theorem 4.3, if $\delta_1, \delta_2, \delta_3$ form a monotone sequence, then let $\chi(a_1, a_2, a_3, a_4) = \phi(\delta_1, \delta_2, \delta_3)$. If $i = 2$ is a local maximum, let $\chi(a_1, a_2, a_3, a_4) = 1$. If $i = 2$ is a local minimum, let $\chi(a_1, a_2, a_3, a_4) = 2$. The remaining 4-tuples may be colored arbitrarily. The proof that this coloring verifies (1) follows as in the proof of Lemma 17 in Chapter 4 of [2].

Acknowledgement. We would like to thank Asaf Shapira and Guy Moshkovitz for pointing out that Theorem 4.3 is incorrect as stated in the case $k = 4$.

References

[1] J. Fox, J. Pach, B. Sudakov, and A. Suk Erdős-Szekeres-type theorems for monotone paths and convex bodies, *Proc. London Math. Soc.* **105** (2012), 953–982.

[2] R. L. Graham, B. L. Rothschild, and J. H. Spencer, *Ramsey Theory*, 2nd Edition, Wiley, New York, 1990.